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An introduction to the sonar equations with applications

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

AN INTRODUCTION TO THE SONAR EQUATIONS
WITH APPLICATIONS

A. B. Coppens and J. V. Sanders

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al Postgraduate School
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Underwater Sound Propagation	
Transmission Loss	

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report provides an introduction to the SONAR equations for those interested in underwater sound as applied to ASW but lacking either the mathematical background or the time for a more rigorous presentation. Earlier versions of these notes were developed for Continuing Education courses presented at Moffett Field, California, and Naval Torpedo Station, Washington. Additionally, these notes have been in demand for certain courses at the Naval Postgraduate School. While this is the text for these courses and

20. should be supplemented by lectures, we have attempted to design the material so that it is reasonably self-explanatory, communicating many of the essential concepts without requiring extensive verbal amplification. The unusual format has been deliberately chosen to facilitate these goals, and our experiences in presenting these materials have seemed to justify this choice. It is assumed that the reader has some familiarity with trigonometric functions and either has or will develop with the aid of the appendix the facility of handling scientific notation and logarithmic operations.

TABLE OF CONTENTS

One example of a SONAR equation, for the passive detection of a target which is radiating sound into the ocean, is

$$EL - TL \geqslant NSL + 10 \log w - DI + DT$$

where

EL = Source Level of the target being detected passively

TL = Transmission Loss as the signal propagates to the detector

NSL = Noise Spectrum Level of the ambient noise in the ocean

w   = Frequency Bandwidth of the detecting system

DI = Directivity Index of the detecting system

DT = Detection Threshold of the detecting system for a specified probability of detection and a specified false-alarm rate

The purpose of this course is to provide the physical and conceptual background necessary to understand the meaning of each of these terms and how they are combined to form the SONAR equations. At the end of this course, the student should be able to apply the appropriate SONAR equation to a given problem and estimate such things as maximum detection range, optimum receiver depth, necessary bandwidth, etc.
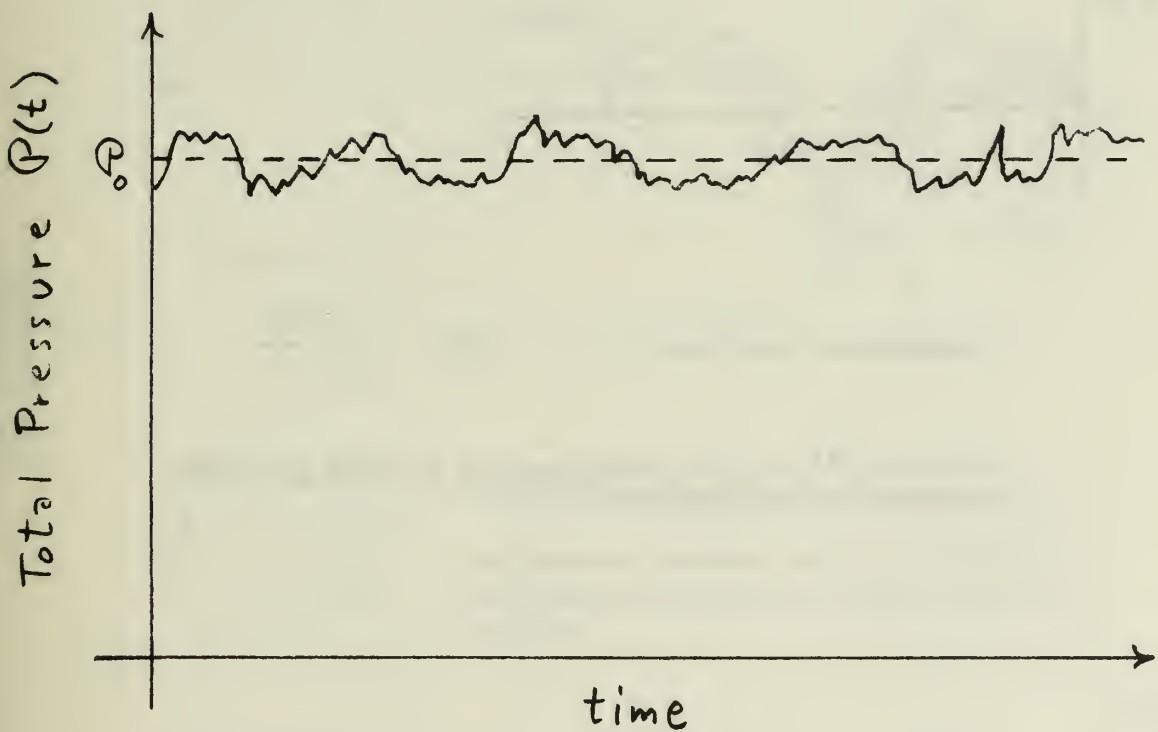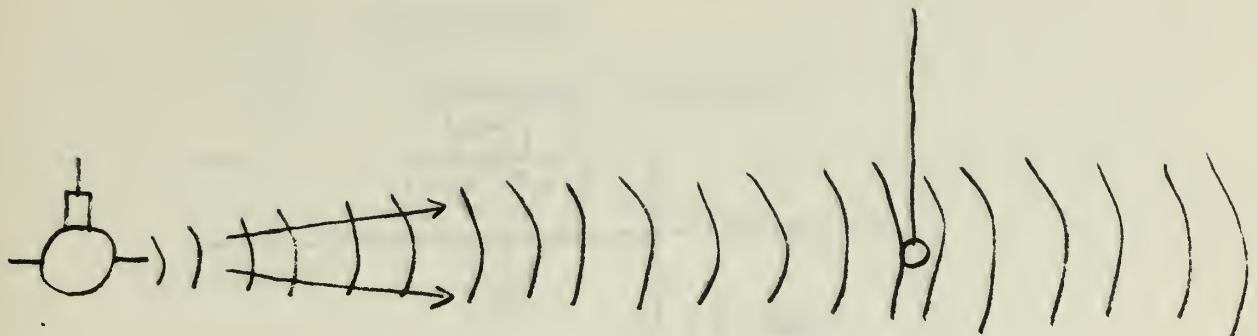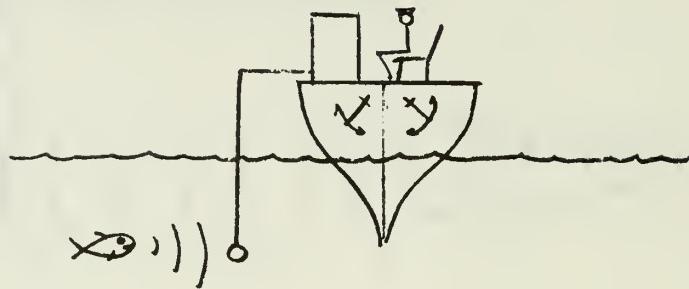
Acoustic Pressure $= P(t) - P_o = p(t)$.

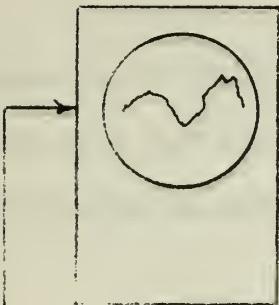$p(t)$ is the quantity sensed by most hydrophones

Hydrophone sensitivity $\qquad \mathcal{M} \equiv \dfrac{v(t)}{p(t)}$

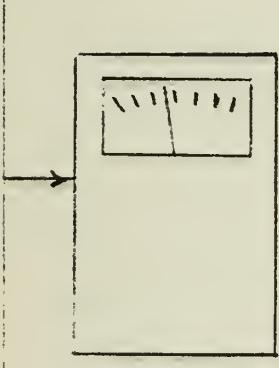where $v(t)$ is the instantaneous voltage at output terminal of hydrophone

DISPLAYS

OSCILLOSCOPE

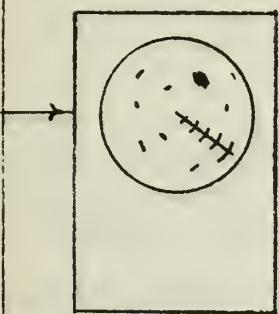instantaneous voltage v(t)
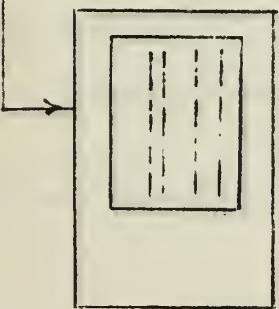
VOLTMETER

some average voltage $\frac{1}{T}\int_{0}^{T}|v|\,dt$

or

$$\sqrt{\frac{1}{T}\int_{0}^{T}v^{2}\,dt}$$

BEARING-RANGE RECORDER (active sonar)

brightness related to the strength of
the echo returned from each range and
bearing

LOFAR-GRAM (passive sonar)

paper is electrically burnt, with
darkness related to the strength of
the signal received at each frequency
and time

from hydrophone
via processor

For ease of discussion, we will at first restrict ourselves to consideration of tonals (monofrequency signals). Later on, we will see that more complicated signals can be broken down by FFT, Fourier Analysis, or other filtering techniques into a collection of monofrequency signals.

The time history of the acoustic pressure for a monofrequency signal can be represented as
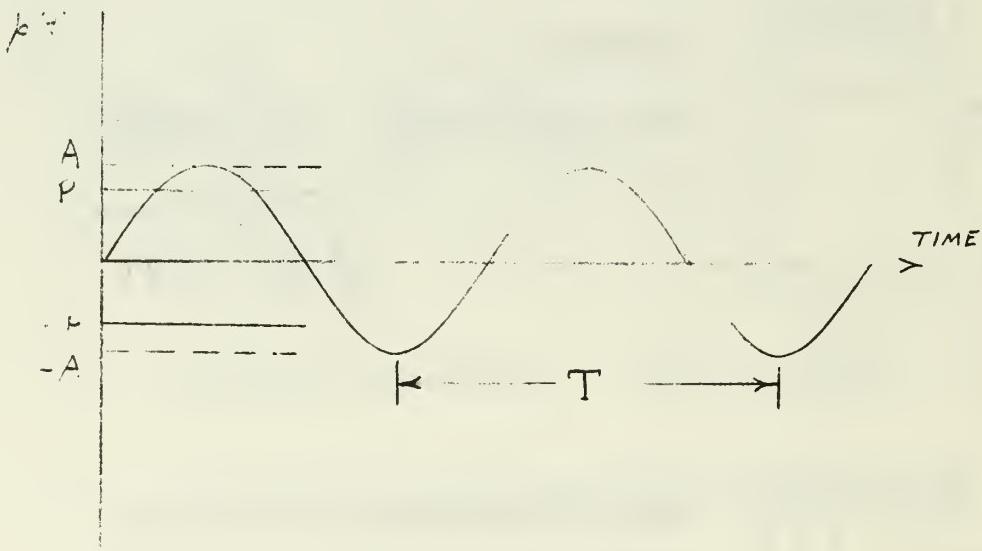
P = The effective amplitude of the acoustic pressure. This is not the same as the peak amplitude A, but is more convenient. We will never use the peak amplitude, but it turns out that P = 0.707A for the above signal.
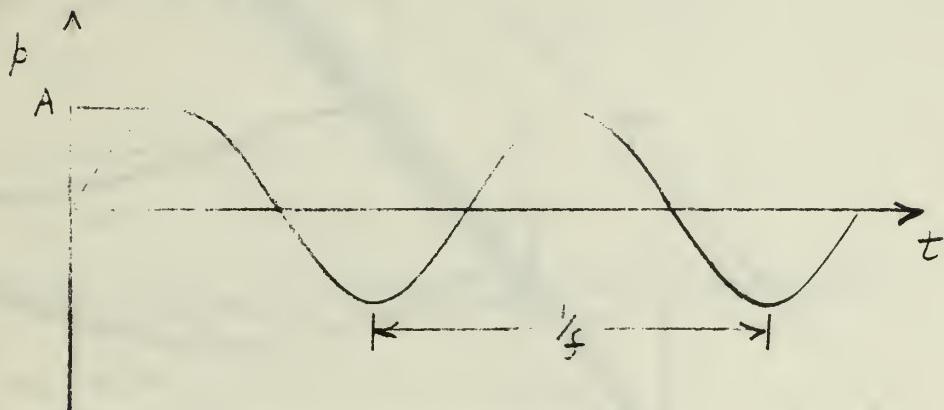
T = The period of the monofrequency signal. The period is the time interval required for the acoustical signal to go through one complete cycle, returning to the configuration it had at the beginning of the interval.

6

PLANE WAVES

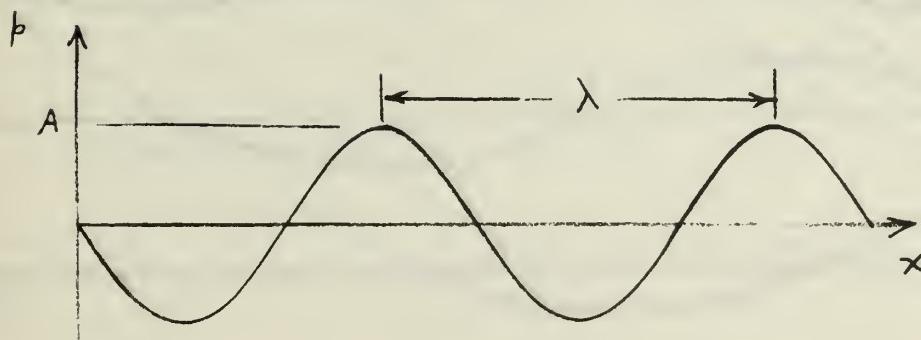$$p = A \sin 2\pi f\left(t - \frac{x}{c}\right)$$

If   x = constant   (For simplicity, take x = 0.)

$$p = A \sin(2\pi f t)$$

If   t = constant   (For simplicity, take t = 0.)

$$p = -A \sin\left(\frac{2\pi}{\lambda} x\right)$$

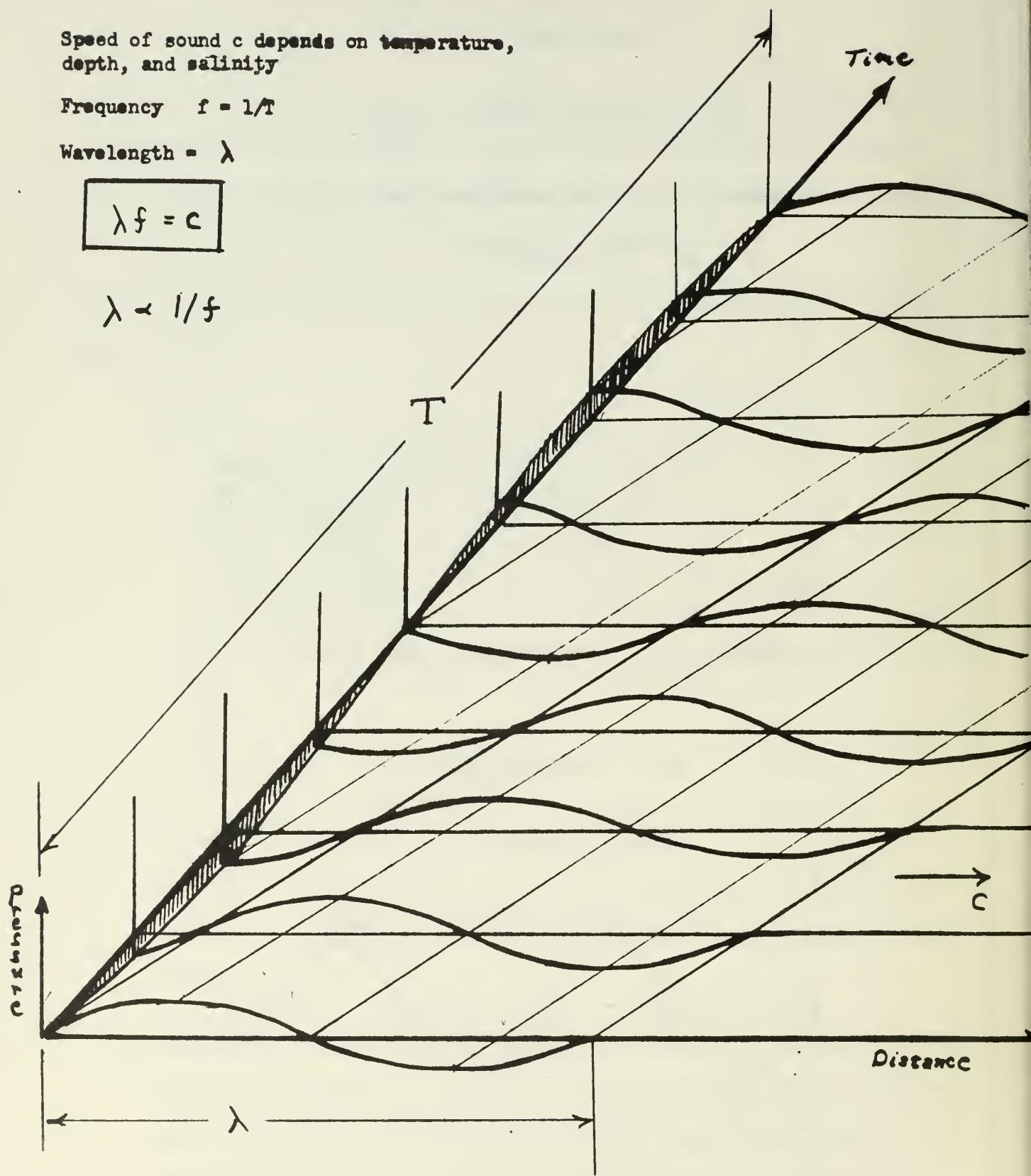As time increases, the wave moves towards larger x.

A WAVE IN SPACE AND TIME

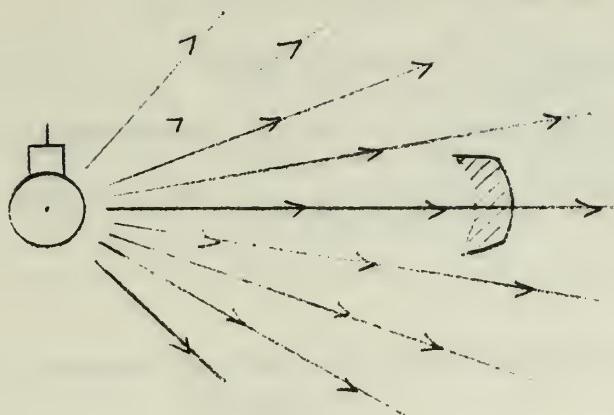Speed of sound c depends on temperature, depth, and salinity

Frequency $f = 1/T$

Wavelength $= \lambda$

$$\lambda f = c$$

$$\lambda \propto 1/f$$

Intensity = power per unit area

where, "area" is the area of a "window" perpendicular
to the direction of travel of the wave.

In terms of measureable quantities,

$$\text{Intensity} = \frac{P^2}{\rho c}$$

where ρc = specific acoustic impedance of the fluid

ρc = 1.5×10^6 Rayl   for water

ρc = 415 Rayl   for air

1 Rayl = $1 \text{ kg/m}^2 \text{sec}$

SIZES OF THINGS IN WATER

Speed of sound, frequency, and wavelength:

The speed of sound in sea water is approximated by

$$c \sim 1.5 \times 10^3 \text{ m/sec} \pm \text{ a few per cent.}$$

It must be remembered that because of depth, temperature, salinity, and contamination there may be variations up to a few per cent in the exact value of c.

Approximate sizes of wavelengths of monofrequency sounds of different frequencies are:

Frequency	Wavelength
10 Hz = 10 cps	150 m
50	30
100	15
500	3
1 kHz = 1000 cps	1.5
5	0.3
10	0.15
50	0.03 = 3 cm

$$1 \text{ m } = 1.09 \text{ yd}$$

Pressure amplitudes:

$\mathcal{P}_o$ = 1 atmosphere = 14.7 lb/in^2 at the ocean surface

$\phantom{\mathcal{P}_o}$ = 10^6 µb $\quad$ (µb = microbar)

$\phantom{\mathcal{P}_o}$ = 10^{11} µPa $\quad$ (µPa = microPascal)

$\mathcal{P}_o$ increases by about 1 atm for every 33 ft of depth in water.

Representative acoustic pressures generated near a sonar transmitter:

$$P \sim 10^4 \text{ to } 10^6 \text{ µb or 0.01 to 1 atm.}$$

If $P \sim \mathcal{P}_o$, then <u>cavitation</u> is likely and performance may be degraded.

Representative weak acoustic pressures detectable by a sonar receiver:

$$P \sim 10^{-2} \text{ µb or } 10^{-8} \text{ atm.}$$

SOUND PRESSURE LEVEL

$$\boxed{SPL = 20 \log(P/P_{ref}) \text{ re } P_{ref}}$$

$$P_{ref} = \begin{cases} 0.0002 \ \mu b & \text{(old noise measurements)} \\ 1\mu b & \text{(old conventional standard)} \\ 1\mu Pa & \text{(new Navy standard)} \end{cases}$$

Examples

If $P = 1$ atm $= 10^{6}\mu b$, then

$$SPL = \begin{cases} 194 \text{ dB re } 0.0002\mu b \\ 120 \text{ dB re } 1\mu b \\ 220 \text{ dB re } 1\mu Pa \end{cases}$$

If $P = 10^{-4}$ atm $= 10^{2}\mu b$, then

$$SPL = \begin{cases} 114 \text{ dB re } 0.0002\mu b \\ 40 \text{ dB re } 1\mu b \\ 140 \text{ dB re } 1\mu Pa \end{cases}$$

If $P = 10^{-8}$ atm $= 10^{-2}\mu b$, then

$$SPL = \begin{cases} 34 \text{ dB re } 0.0002\mu b \\ -40 \text{ dB re } 1\mu b \\ 60 \text{ dB re } 1\mu Pa \end{cases}$$

CONVERSION BETWEEN SPL's WITH DIFFERENT REFERENCE PRESSURES

For a <u>given</u> pressure amplitude

      SPL re 1μPa = SPL re 1μb + 100 dB

      SPL re 0.0002μb = SPL re 1μb + 74 dB

Notice

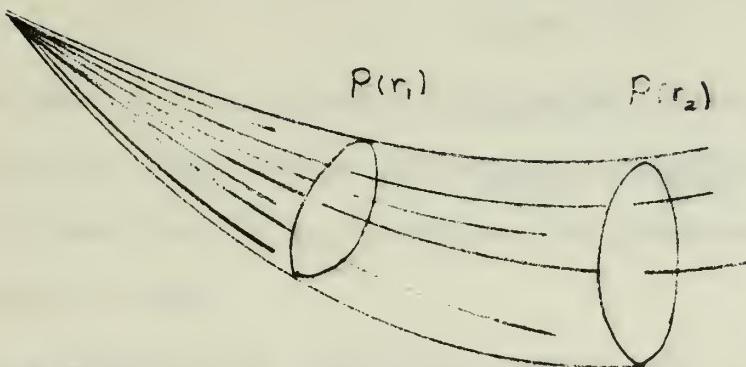0 dB does <u>not</u> mean the absence of sound , **but means** $P = P_{ref}$.

If $P = P_{ref}$,
then

$$SPL = 20 \log (P/P_{ref}) = 20 \log(P_{ref}/P_{ref}) = 20 \log 1 = 0 \text{ dB re } P_{ref}$$

TRANSMISSION LOSS

$$SPL(r_1) = 20 \log \frac{P(r_1)}{P_{ref}}$$

$$SPL(r_2) = 20 \log \frac{P(r_2)}{P_{ref}}$$

$$SPL(r_1) - SPL(r_2) = 20 \log \frac{P(r_1)}{P(r_2)}$$

$$TL = SPL(1 \text{ meter}) - SPL(r \text{ meters}) = 20 \log \frac{P(1)}{P(r)}$$

SAMPLE CALCULATION OF TRANSMISSION LOSS

If SPL at 1 meter = 120 dB re 1μb

and SPL at range r = 40 dB re 1μb

then TL = 120 - 40 dB = 80 dB

TL does not depend on the reference pressure. Let's convert the above SPL's
to reference 1μPa and recalculate the TL

Now SPL at 1 meter = 220 dB re 1μPa

and SPL at range r = 140 dB re 1μPa

so that TL = 220 - 140 = 80 dB

TRANSMISSION LOSS FROM SPREADING

The total transmission loss TL can be considered to arise from two different causes:

1) As the rays of sound propagate out from the source and travel through the water, they will bunch together or spread out depending on the properties of the speed of sound profile for the water. The transmission loss which arises from this effect is termed
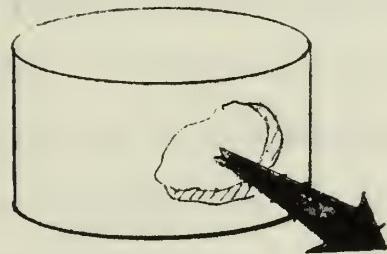
TL_g = transmission loss from spreading

Some simple examples of spreading for rays traveling straight lines are
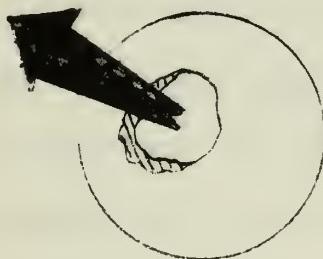
Spherical spreading
$TL_g = 20 \log r$

Cylindrical spreading
$TL_g = 10 \log r$

No spreading
$TL_g = 0$ dB

In the real ocean, the rays never travel in straight lines, so that these above simple examples are unrealistic. We will see, however, that there are many situations in which the sound will spread out spherically or cylindrically over large portions of its path, and the above simple equations will appear in the complete transmission loss equation.

SPHERICAL AND CYLINDRICAL WAVES

SPHERICAL
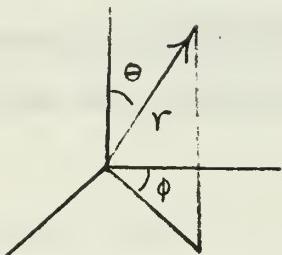
$$p = \frac{A}{r} \sin 2\pi f \left(t - \frac{r}{c} \right)$$

CYLINDRICAL

$$p = \frac{B}{r} \sin 2\pi f \left(t - \frac{r}{c} \right)$$

$$IF \quad r \gg \lambda$$

TRANSMISSION LOSS FROM DISSIPATION

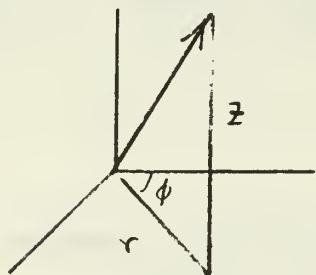2) There are a variety of other physical effects termed <u>dissipative</u>
effects, which reduce the SPL of the sounds in addition to the spreading
of the rays. Since the spreading of rays is described by TL_g, we will call
the portion of the transmission loss which results from these other effects
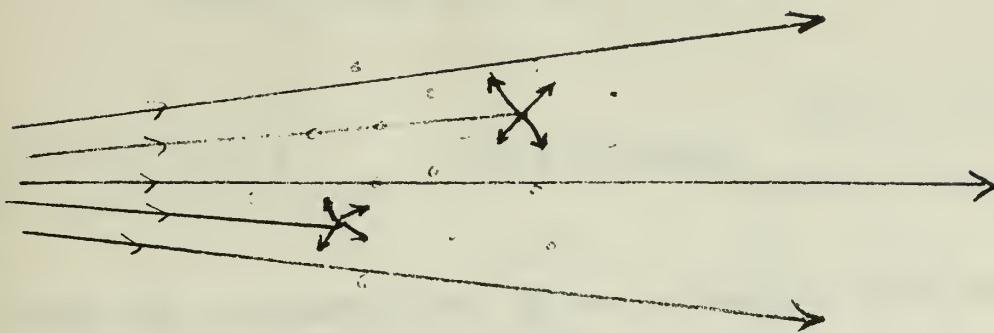
TL_ℓ = Transmission Loss from dissipation

There are a number of these which are important in underwater sound propagation:

(a) Absorption. As sound travels through the water, there is a
conversion of the acoustical energy into thermal energy because
of effects which are very similar to friction in moving machinery
and hysteresis losses in transformers.

(b) Transmission into the bottom: When a sound beam strikes the ocean
bottom, part of the beam is reflected back upward in the water
and another part is sent down into the bottom. The beam which
is reflected back upward in the water **contains less energy**
than the beam which traveled down to the bottom.

(c) Scattering. The presence of rough boundaries at the ocean surface
and bottom can lead to a scattering of the sound out of the reflected
beam. The presence of bubbles and other impurities in the water
itself can also scatter sound energy out of the path of the ray.

(d) Leakage. In certain kinds of sound propagation in the ocean,
like the surface layer, the rays of sound can "leak" out the
bottom of the layer each time they bend down to graze it. This
"leakage" results in a loss of energy from the layer.

ABSORPTION IN SEA WATER

$TL_\ell = a \pi$

Attenuation Constant (dB/m)

Frequency
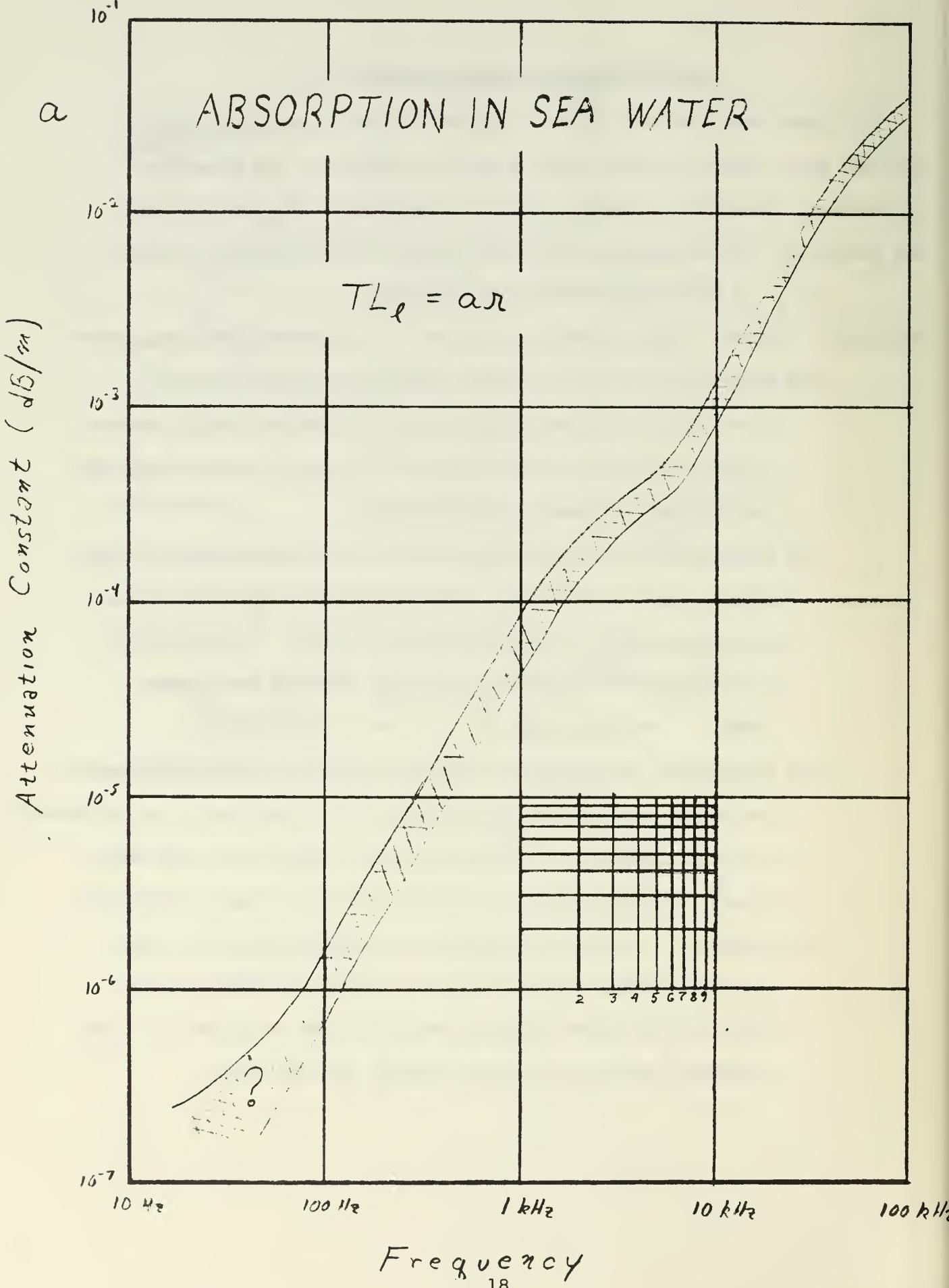
OTHER LOSS MECHANISMS

Volume Scattering

Surface Scattering (top + bottom)

Transmission into bottom with scattering
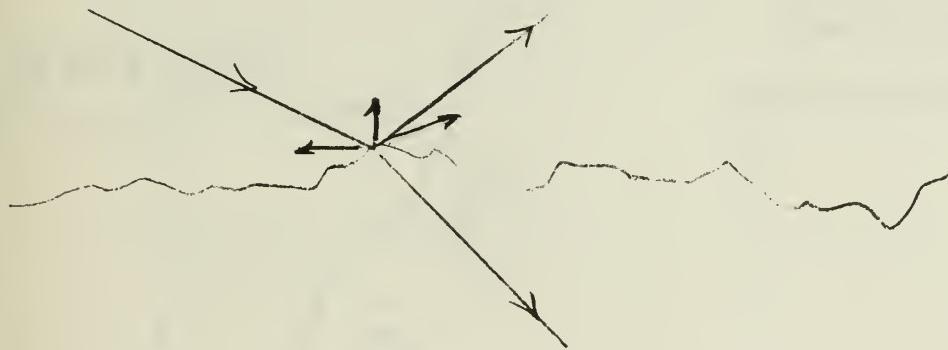
Leaky Duct with scattering

19
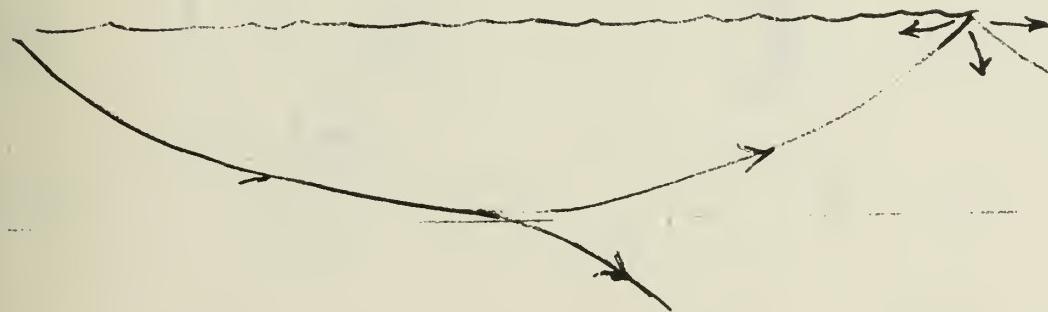
TRANSMISSION LOSS

In general the total transmission loss TL can be composed of two terms,

$$TL = TL_g + TL_\ell$$

where $\qquad TL_g$ = transmission loss from geometrical spreading

$\qquad\qquad\quad TL_\ell$ = transmission loss from dissipation

SPEED OF SOUND, TEMPERATURE, AND DEPTH
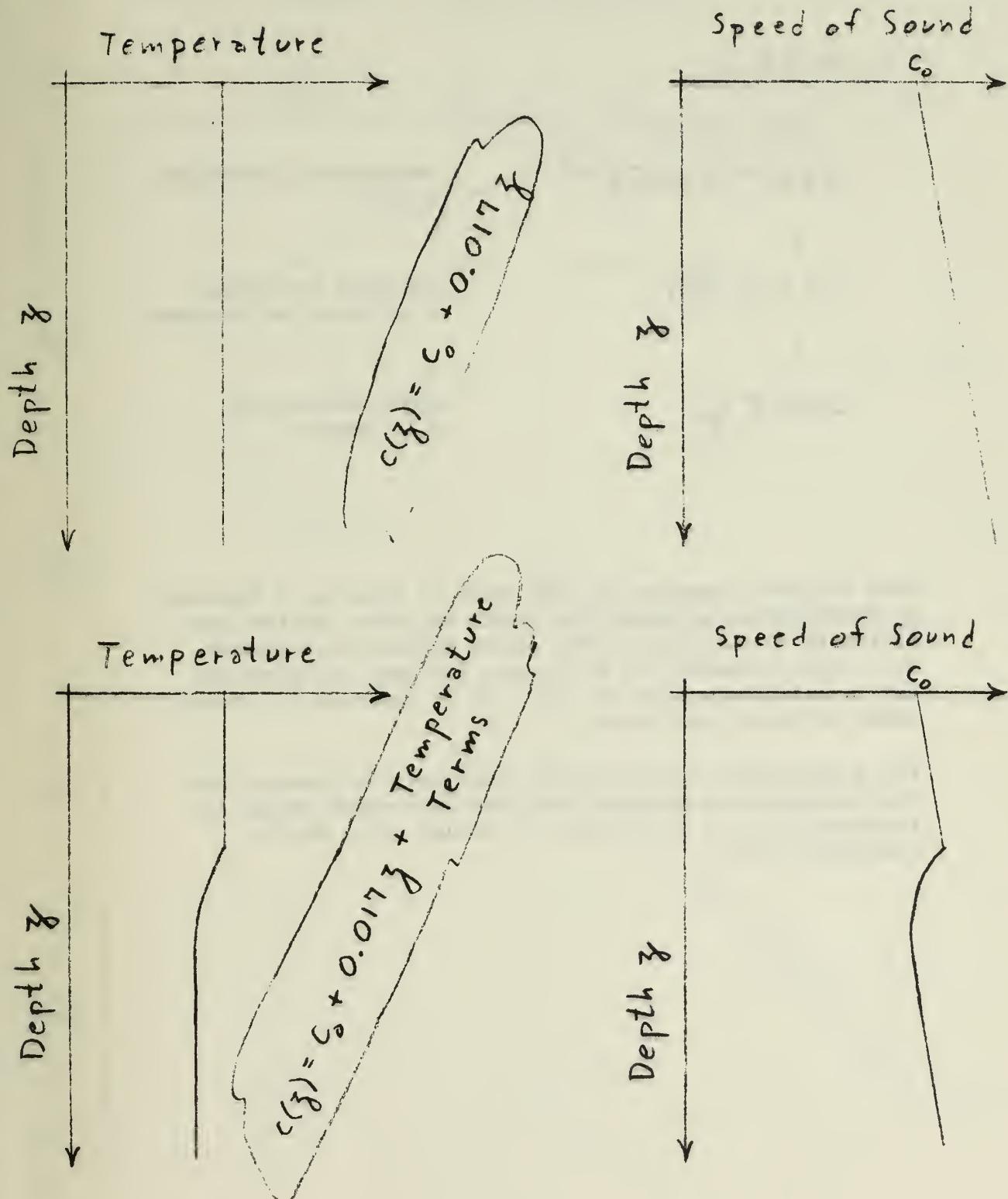
These curves show how the speed of sound is related to depth and temperature. The effect of salinity is not included, so these curves are appropriate only for water of constant salinity.

$$c \approx 1449.2$$

$$+$$

$$4.62\,T - 0.55\,T^2 \qquad \text{Temperature Correction} \\ \text{T in } ^\circ\text{C}$$

$$+$$

$$1.3\,(S-35) \qquad \text{Salinity Correction} \\ \text{S in parts per thousand}$$

$$+$$

$$0.017\,z \qquad \text{Depth Correction} \\ \text{z in meters}$$

Very accurate formulas for the speed of sound as a function of temperature, salinity, and depth are very involved and difficult to calculate. The above expression, although not highly accurate, is sufficient for many calculations and is relatively easy to use. It is accurate to within about 0.1 meter per second.

For a discussion of many of the more precise formulas and the discrepancies between them, the interested reader is referred to V. A. Del Grosso, J. Acoust. Soc. Am. <u>56</u>, 1084-1091 (1974).

TYPICAL SOUND SPEED PROFILE

Speed of Sound (m/sec)

1470   1480   1490   1500   1510   c

surface layer

Seasonal thermocline

Main thermocline

1000 — — — — — — — — — — — — sound axis

2000

Deep isothermal layer

3000

0.017 m/sec/m of depth

4000

z
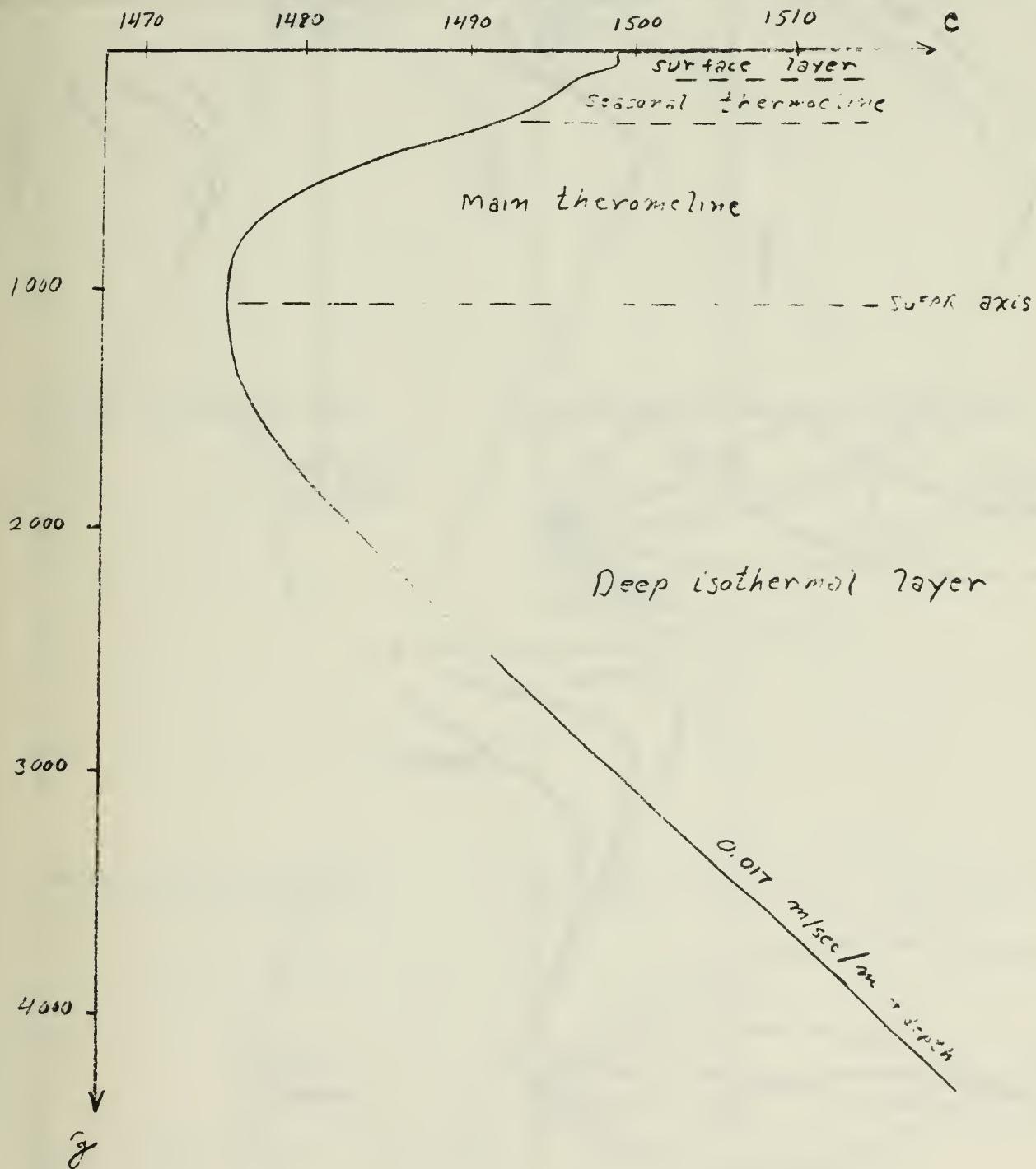
PROFILE VARIATIONS

HOURLY
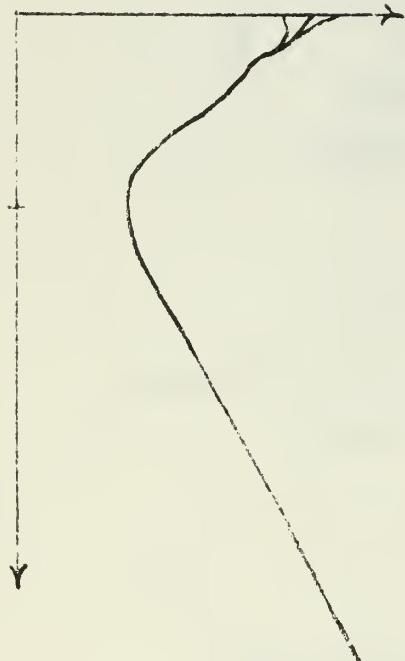
WEEKLY
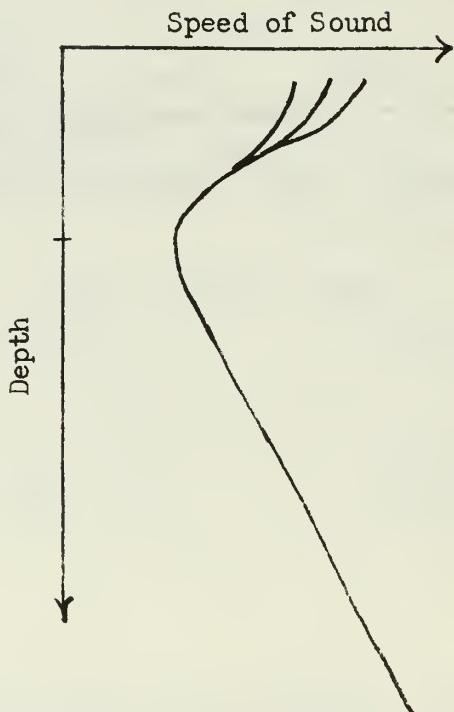
SEASONAL

GEOGRAPHIC

Speed of Sound c(z)

Range r

Negative
Gradient

Depth z

Depth z

Shadow
Zone
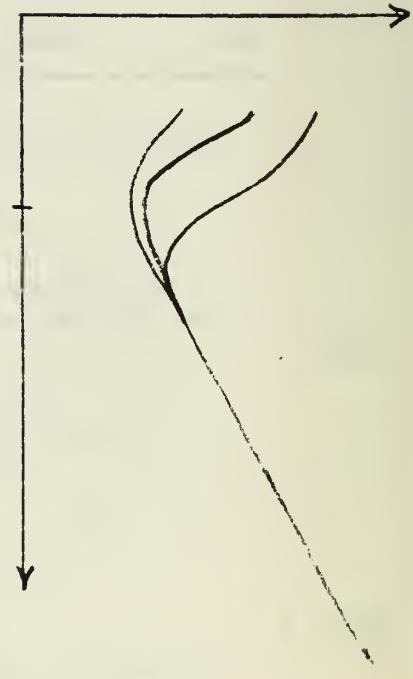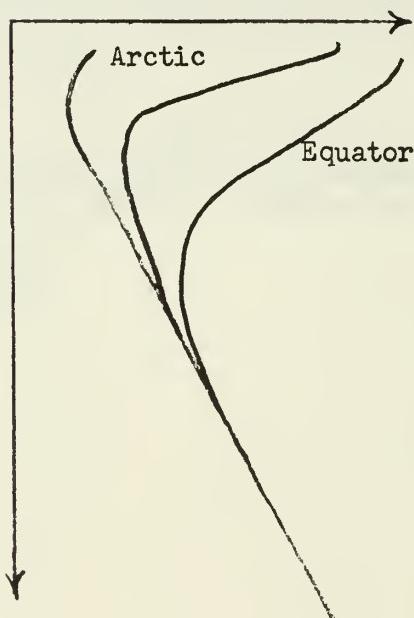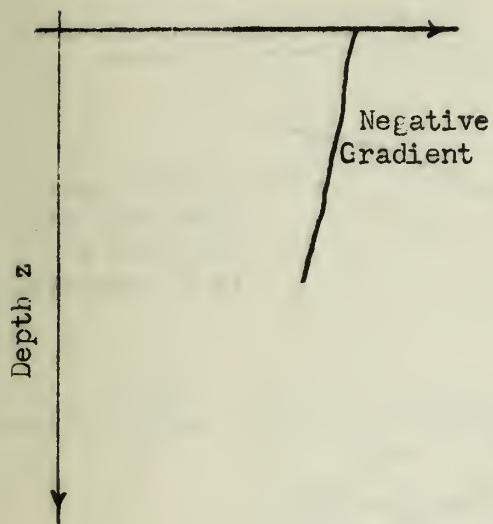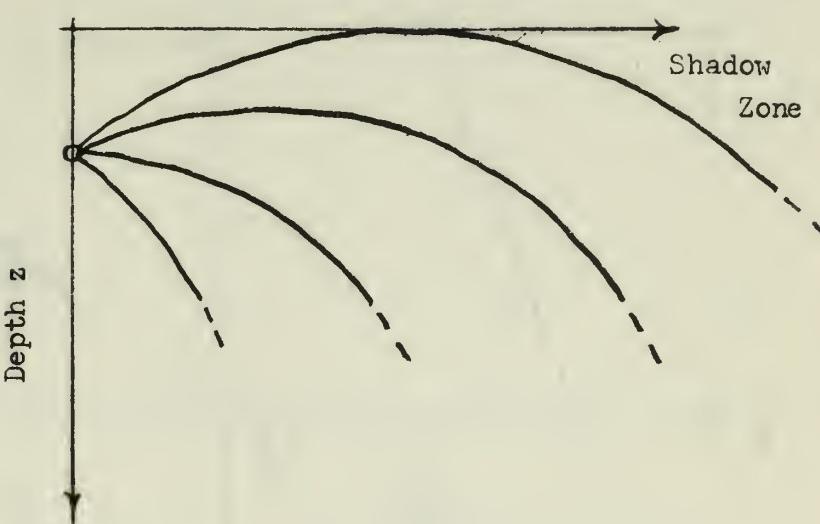
Positive
Gradient

Sound
Channel

$c(z)$

r

c_o

Snell's Law:
Along any ray
$c(z)/\cos \theta$
is a constant.

z

z

θ

R

R is the
radius of curvature
of the ray

$-\dfrac{c_o}{g}$

$c(z) = c_o + g z$	g is the <u>gradient</u>
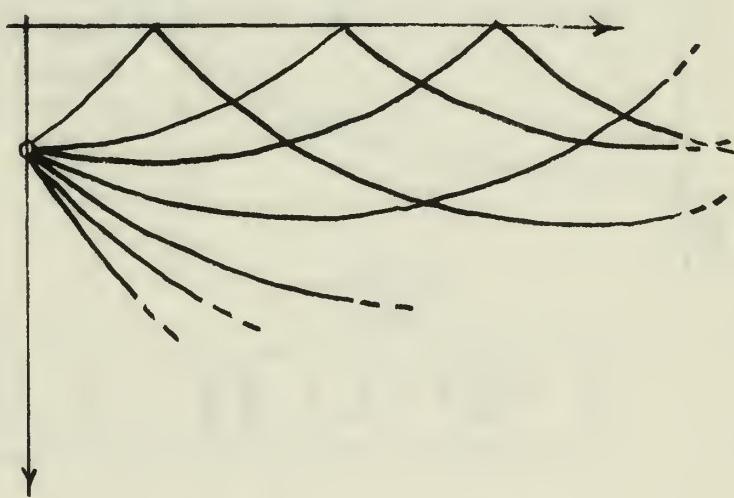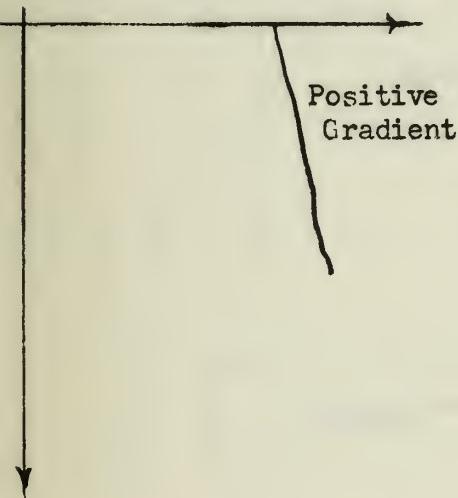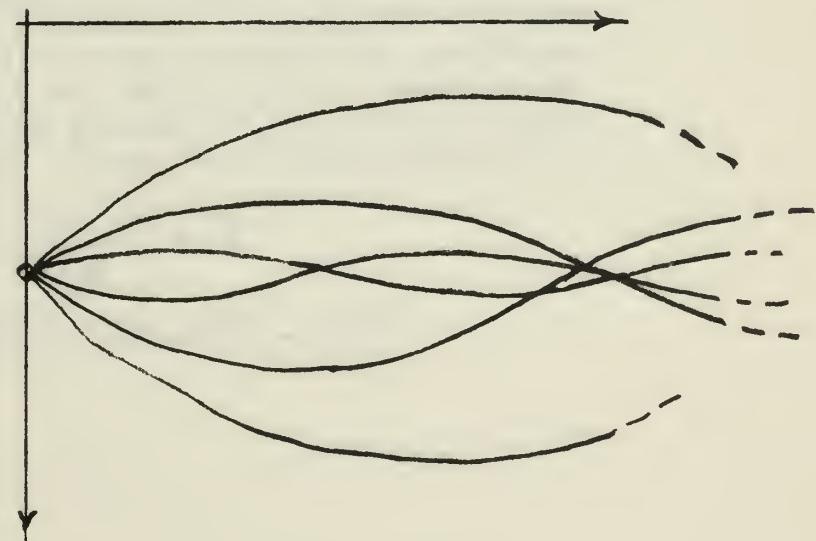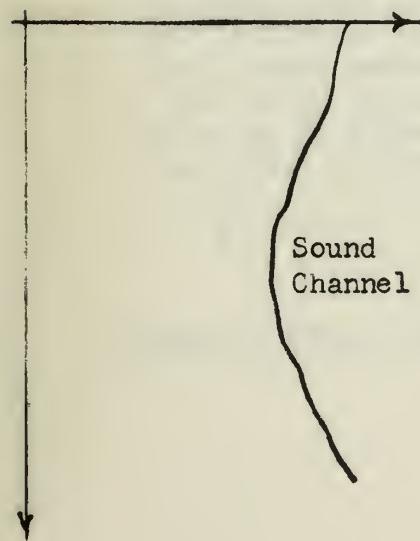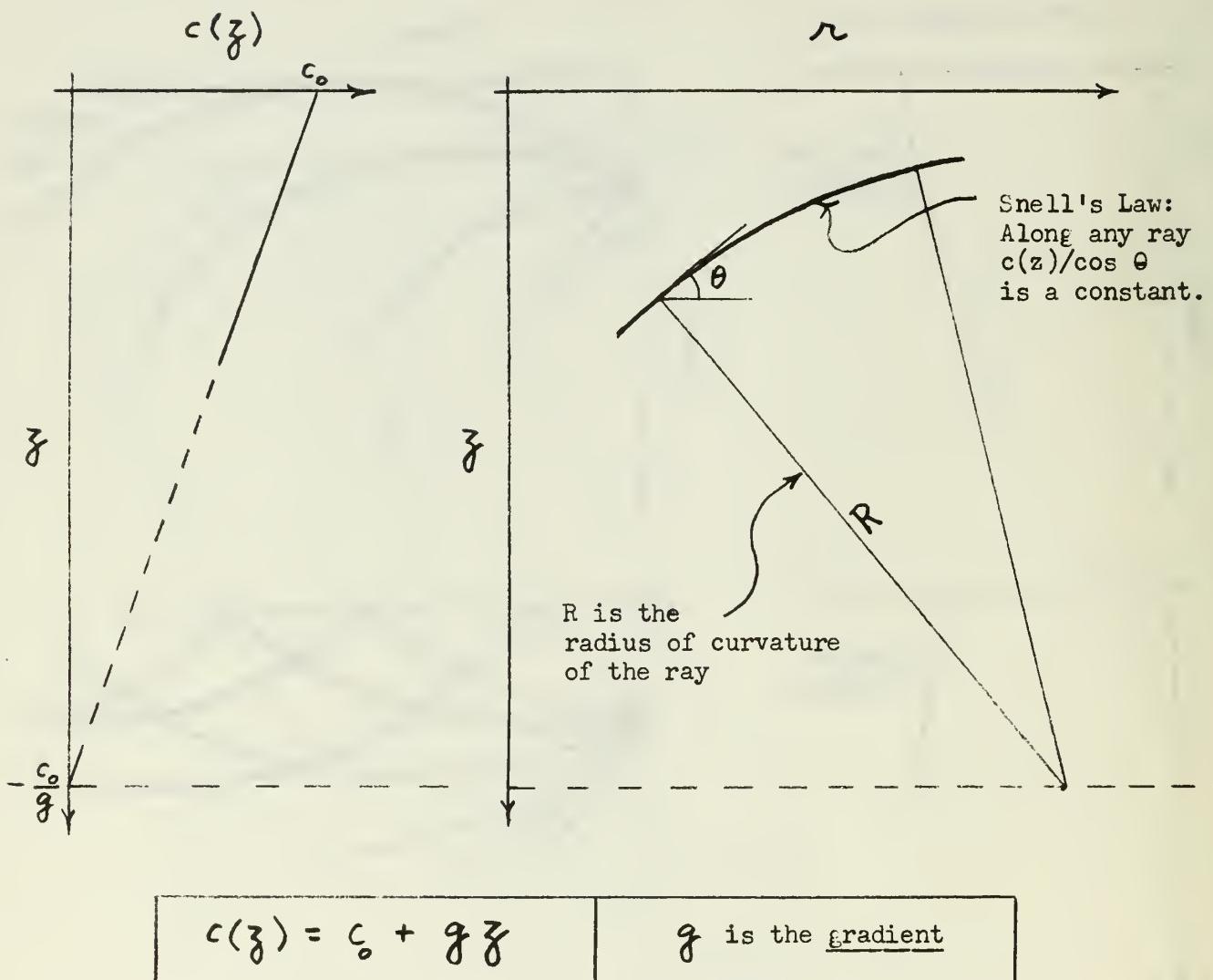
The gradient g can be either positive or negative.
For a speed of sound profile composed of layers within each of
which the gradient has constant value, then the segment of the
ray within each layer is the arc of a circle whose center lies
at the depth $z = -c_o/g$ below the top of the layer (where the
speed of sound is c_o at the top of the layer and g is the value
of the gradient for that layer). The radius of the circle is

$$R = \left| \frac{c}{g \cos \theta} \right| .$$

<u>The ray always bends toward the lower speed of sound.</u>

Line of centers for bottom layer

$c(z)$

r

Top
layer

Bottom
layer

Line of centers for top layer

z
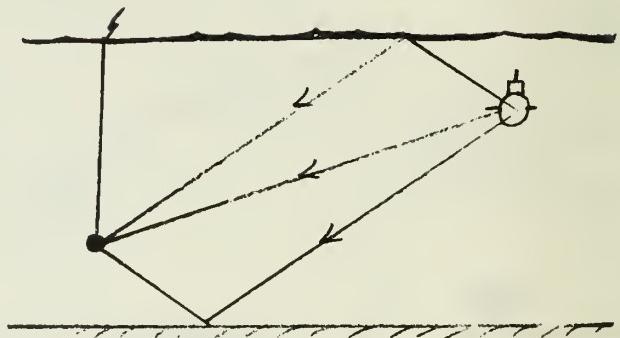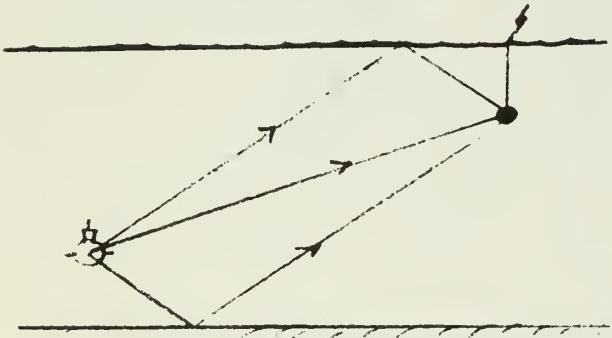
If the direction the sound travels over the propagation paths is reversed by exchanging the positions of the source and receiver, and the propagation conditions remain the same, then the transmission loss between source and receiver will remain the same.

Active
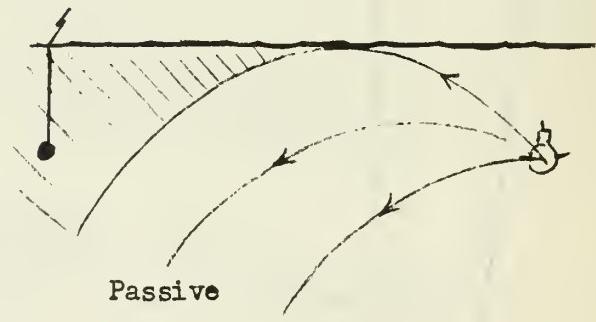
Passive

As a consequence, the transmission loss suffered by the signal in reaching the target from an active source will be identical with the transmission loss experienced by a signal of the same frequency (and duration) radiated from the target when it reaches a receiver.
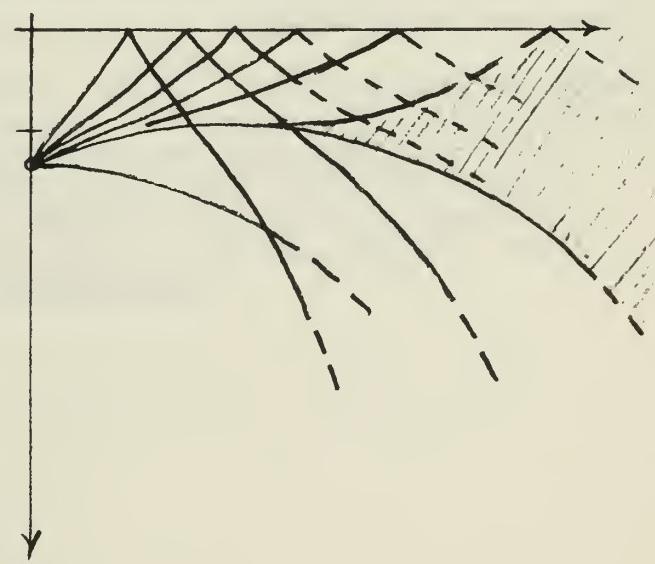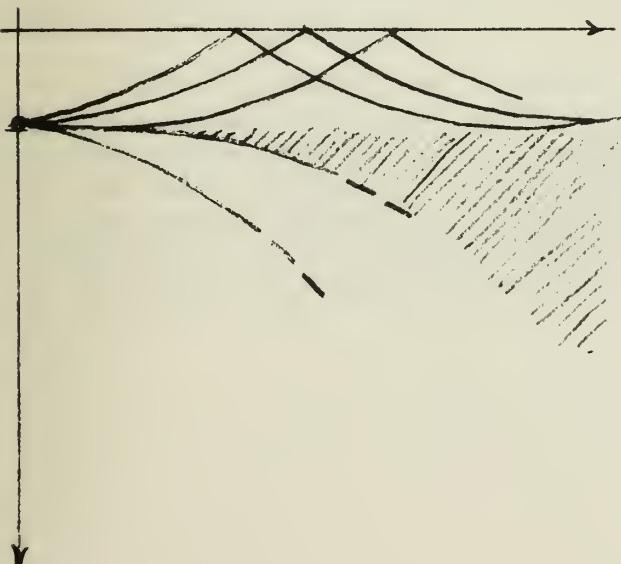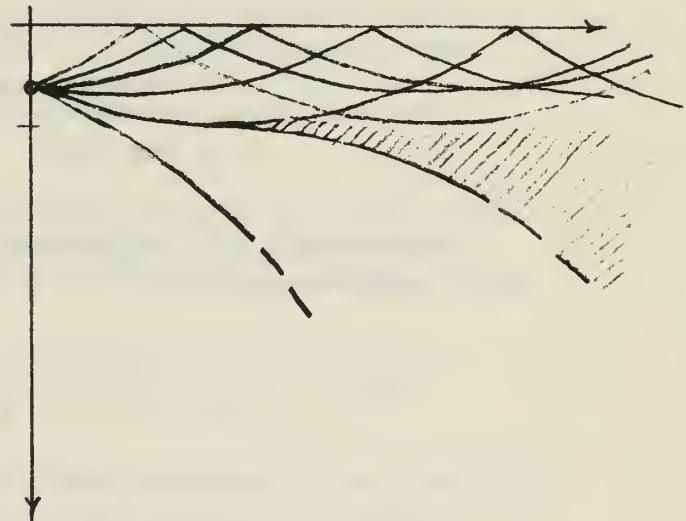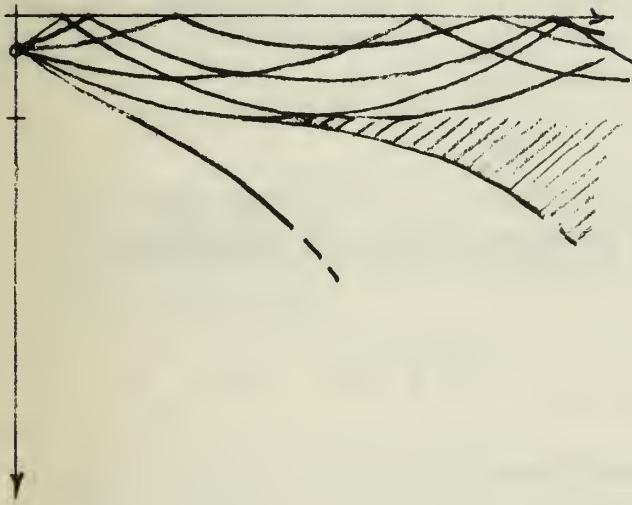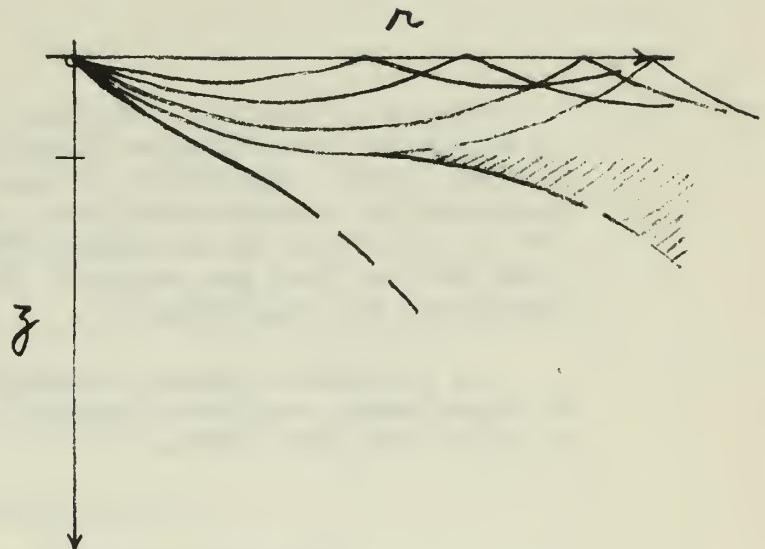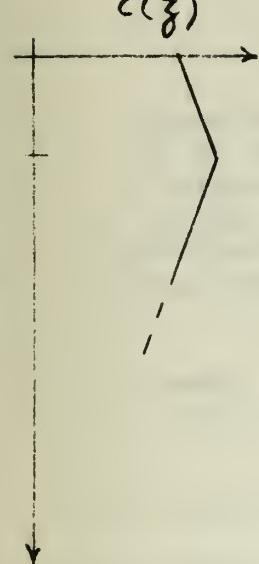
When a ray reflects from the ocean surface, some of the energy
is scattered into different directions and can propagate out of the
surface layer. This energy is thus lost from the channel, which
increases the transmission loss. Additionally, energy can diffuse
out of the bottom of the layer. These effects can be described in
terms of the "loss per bounce". The number of bounces is the range
divided by the skip distance r_s.

An empirical formula, resulting from the analysis of a number
of experiments, has been obtained by W. F. Baker, J. Acoust. Soc.
am. **57**, 1198-1200 (1975)--

$$b = 0.63 \times (1.4)^{SS} \times (f \text{ in } kHz)$$

b = loss per bounce in dB

SS = Sea State

This equation is valid within a restricted range of parameters,
given approximately by

$$2 < SS < 5$$

$$3 \text{ kHz} < f < 8 \text{ kHz}$$

$$80 \text{ ft} < \text{layer depth} < 200 \text{ ft}$$

$$30 \text{ ft} < \text{source depth} < 60 \text{ ft}$$
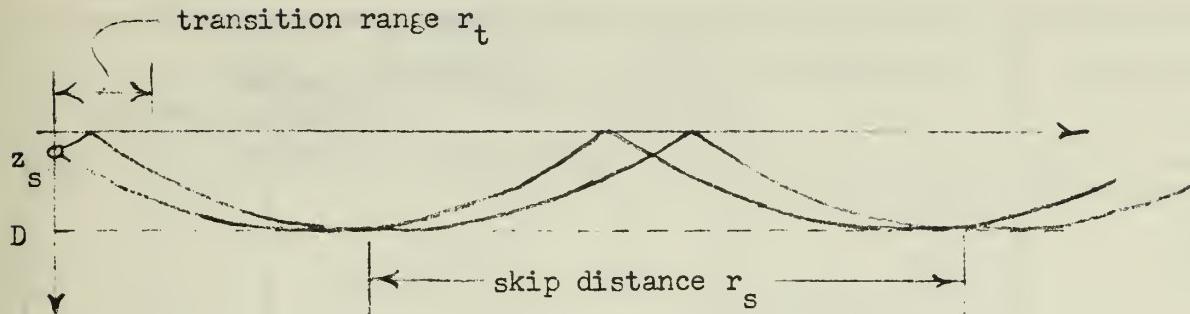
The depth of the mixed layer is designated by D.

The depth of the source in the layer is designated by z_s.

The <u>transition range</u> r_t is the distance the sound beam must travel until it effectively fills the channel. It is calculated from

$$r_t = 1.05 \times 10^2 \sqrt{\frac{D^2}{D - z_s}} \qquad \text{(all distances are in meters)}$$

The <u>skip distance</u> r_s is the distance required for the rays which just graze the bottom of the layer to complete one full cycle from the layer bottom to the surface and back to the bottom. It is calculated from

$$r_s = 8.4 \times 10^2 \sqrt{D} \qquad \text{(all distances are in meters)}$$

If the loss per bounce is designated by <u>b</u> and the energy loss resulting from absorption is designated by a, then the transmission loss for the mixed layer can be estimated by the following formulas:

TL = 20 log r + ar	within transition range
TL = 10 log r_t + 10 log r + ar + br/r_s	beyond transition range

For ranges less than r_t the sound is spreading out <u>spherically</u> to fill the sound channel. For ranges greater than r_t, however, the sound has spread far enough vertically to be reflected from the surface and bent back upward from the bottom of the layer. Once this happens, then the sound is trapped and will spread <u>cylindrically</u>.

REPRESENTATIVE PARAMETERS

Layer depth D	Skip distance r_s	Transition Range r_t		
		$z_s = 20m$	$z_s = 40m$	$z_s = 80m$
25 m	4200 m	1170 m	X	X
50	5930	955	1660 m	X
75	7270	1060	1330	X
100	8400	1170	1360	2350 m

SAMPLE CALCULATION OF A TRANSMISSION LOSS

Assumptions:

Depth of the mixed layer = 40 m
Depth of the source   = 20 m
Frequency of the sound   = 2 kHz
Sea state   = 3

We then determine

Skip distance   = 5300 m
Transition range   = 940 m
Attenuation constant = 0.00015 dB/m
Loss per bounce   = 3.5 dB/bounce

The transmission loss formulas are thus

(a) For ranges less than the transition range,

$$TL = 20 \log r + (1.5 \times 10^{-4})r \quad .$$

(b) For ranges greater than the transition range,

$$TL = 10 \log r + (8.1 \times 10^{-4})r + 30 \quad .$$

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TRANSMISSION LOSS CURVE FOR SOURCE AND RECEIVER IN THE DUCT

Parameters:

      Depth of the mixed layer = 40 m
      Depth of the source   = 20 m
      Frequency of the sound = 2 kHz
      Sea state   = 3
      Receiver is in the duct

———————— Simple model

——— ——— Empirical curve representing
the average of vast amounts of data, from
Navy Underwater Sound Lab. Tech. Mem. 1110-14-55, 1955
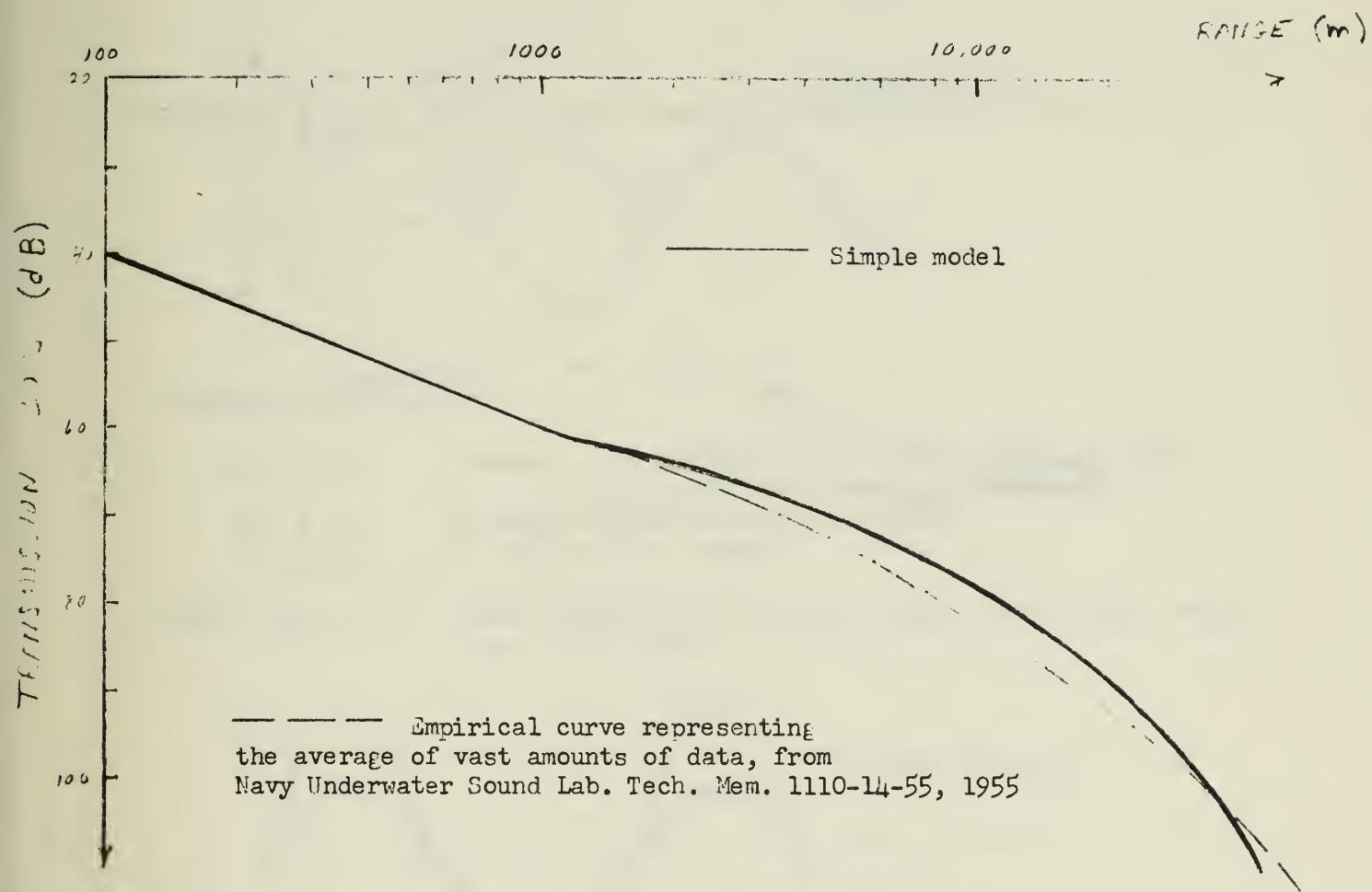
BUT: These results can be radically changed by effects such as
Surface Interference (which becomes important at lower
frequencies and lower sea states), steep gradients below
the layer, and source and/or receiver near the top or
bottom of the duct. Additionally, if the frequency of
the sound is too low, it <u>will not</u> be trapped in the duct.

As a very rough estimate, the frequency must be greater than about
$(2 \times 10^5)/D^{3/2}$, where D is in meters and the frequency is in Hz,
for the sound to be well trapped in the layer and for the above
model to apply.

SURFACE INTERFERENCE
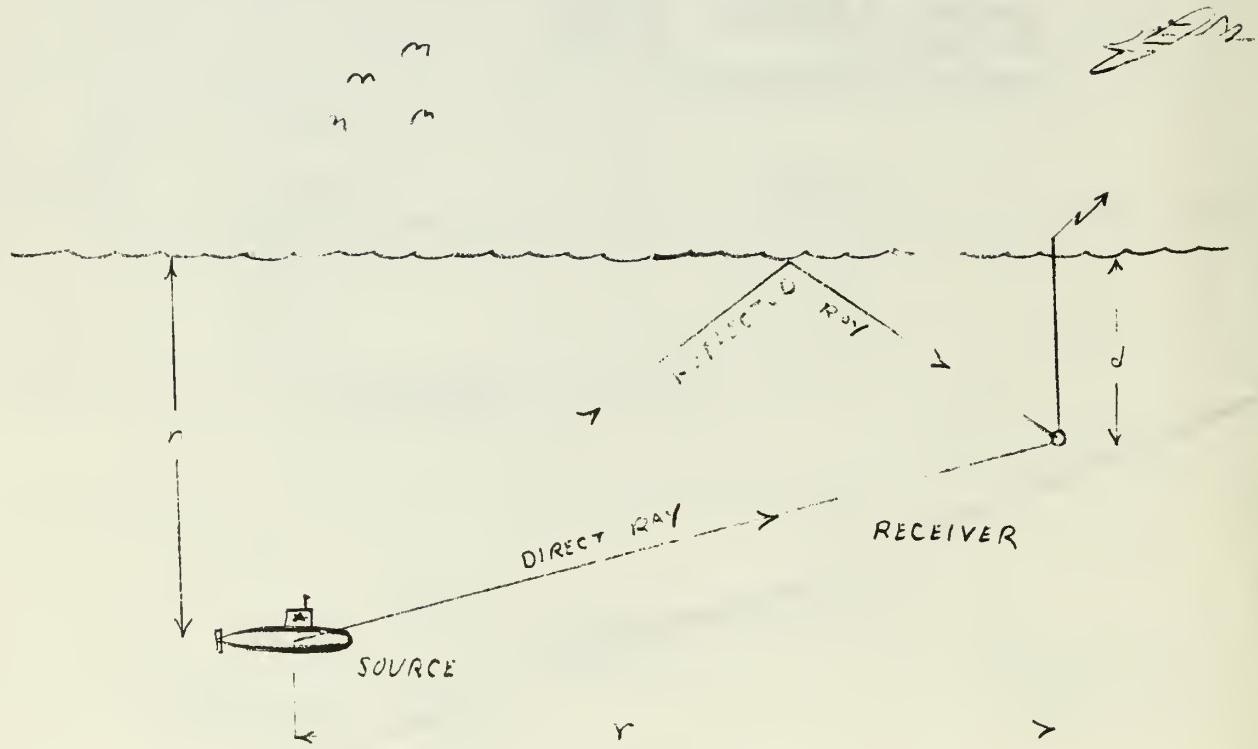
h = SOURCE DEPTH

d = RECEIVER DEPTH

r = RANGE (HORIZONTAL)

Acoustic pressure at the receiver

direct ray
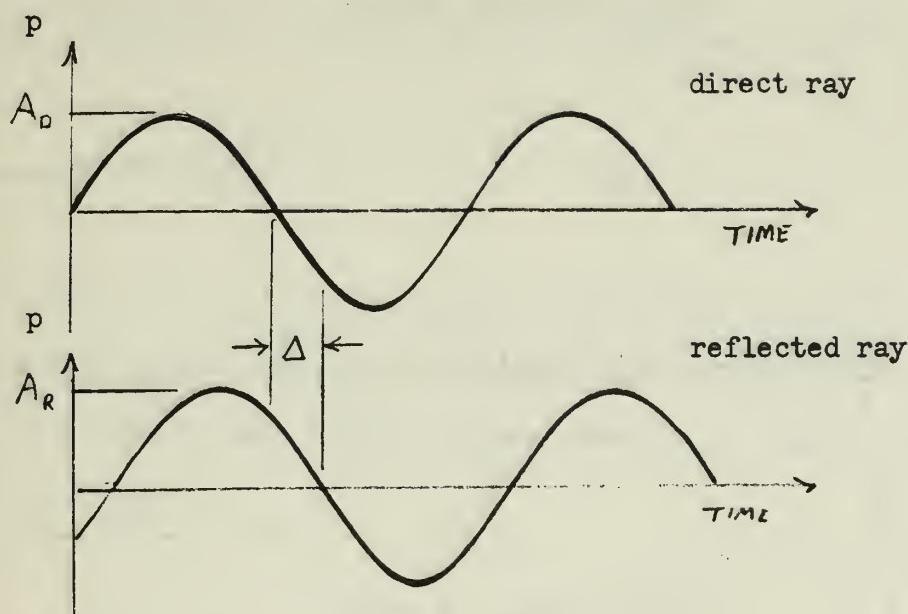
reflected ray

pressure amplitude = A

$A_d > A_r$ because   (1) reflected rays travels a longer path
                                   (2) scattering from rough surface

$A_d \gtrsim A_r$ if range large and surface smooth

$A_d \gg A_r$ if surface very rough

phase delay = Δ (due to difference in path length and to phase change
                             upon reflection)

$\Delta = 0°$

$\Delta = 180°$

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smooth surface $A_d \approx A_r$

rough surface $A_d \gg A_r$

isovelovity ————
ray theory — — —
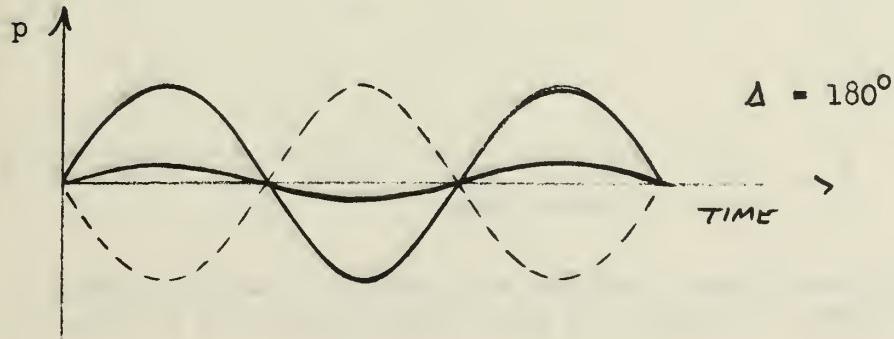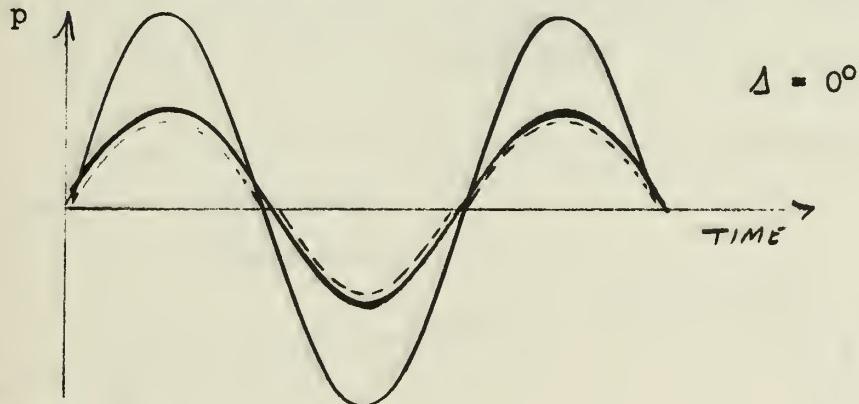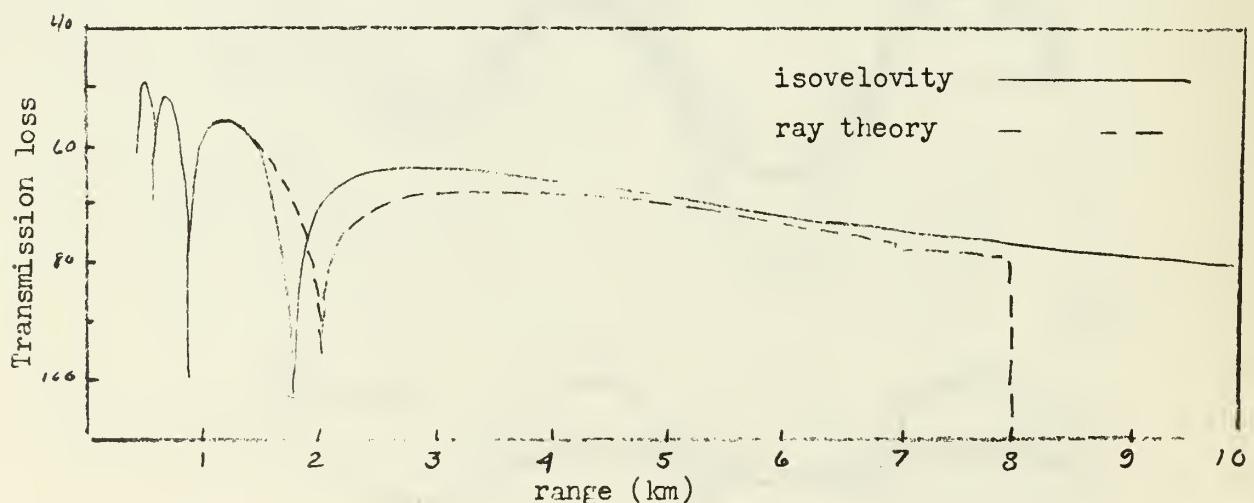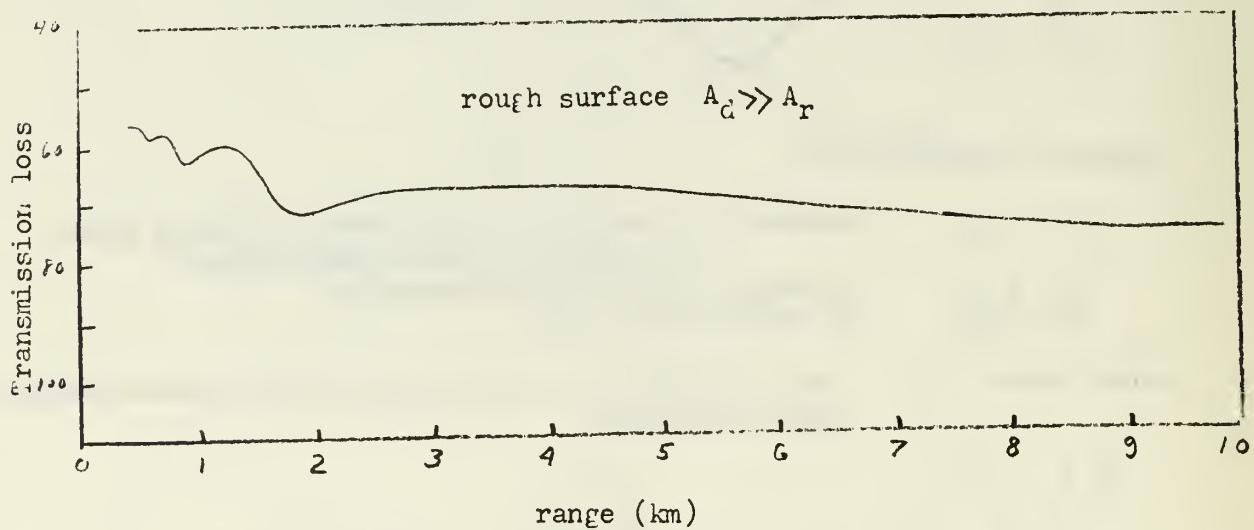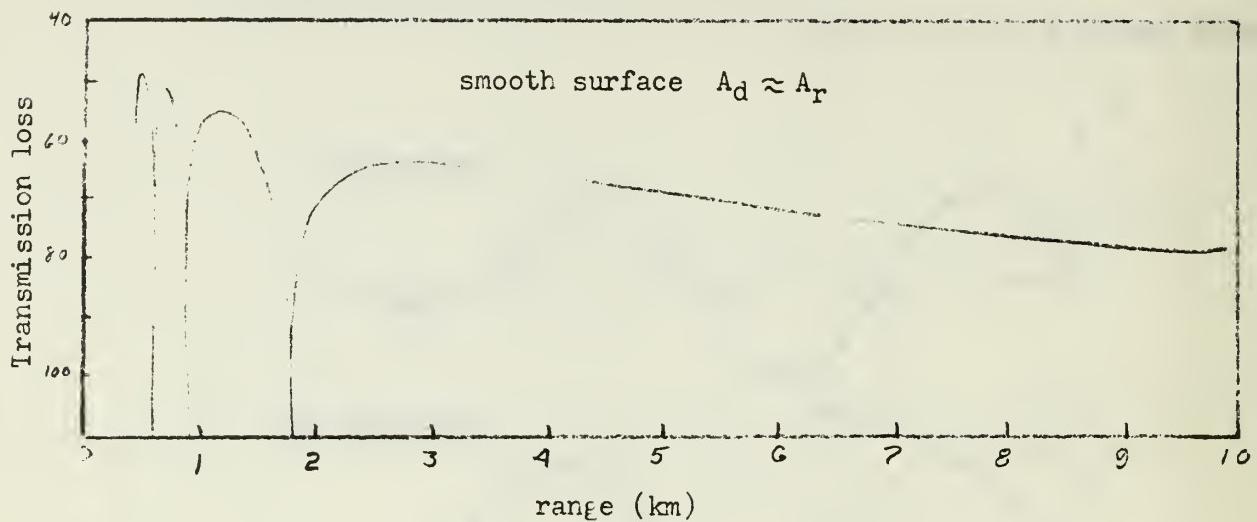
Real sound speed profiles will affect TL only at large
distances. (Notice the shadow zone predicted at 8 km.)

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The range at which the received signal is a minimum is given by

$$r = \frac{2hd}{nc} f \qquad \text{where} \quad n = 1, 2, 3, \cdots$$

It is possible to use information from the received signal to determine the depth, speed, and distance of closest approach of the source. For a source moving at constant speed v along a straight line at a fixed depth h, the range r is given by

$$r^2 = s^2 + v^2 t^2$$
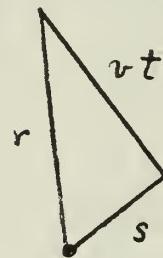
where at t = 0, r = s = the distance of closest approach.

Combining these equations gives the time at which the signal is a minimum as a function of frequency.

$$s^2 + v^2 t^2 = \left(\frac{2hd}{nc}\right)^2 f^2$$

The display that results is shown below:
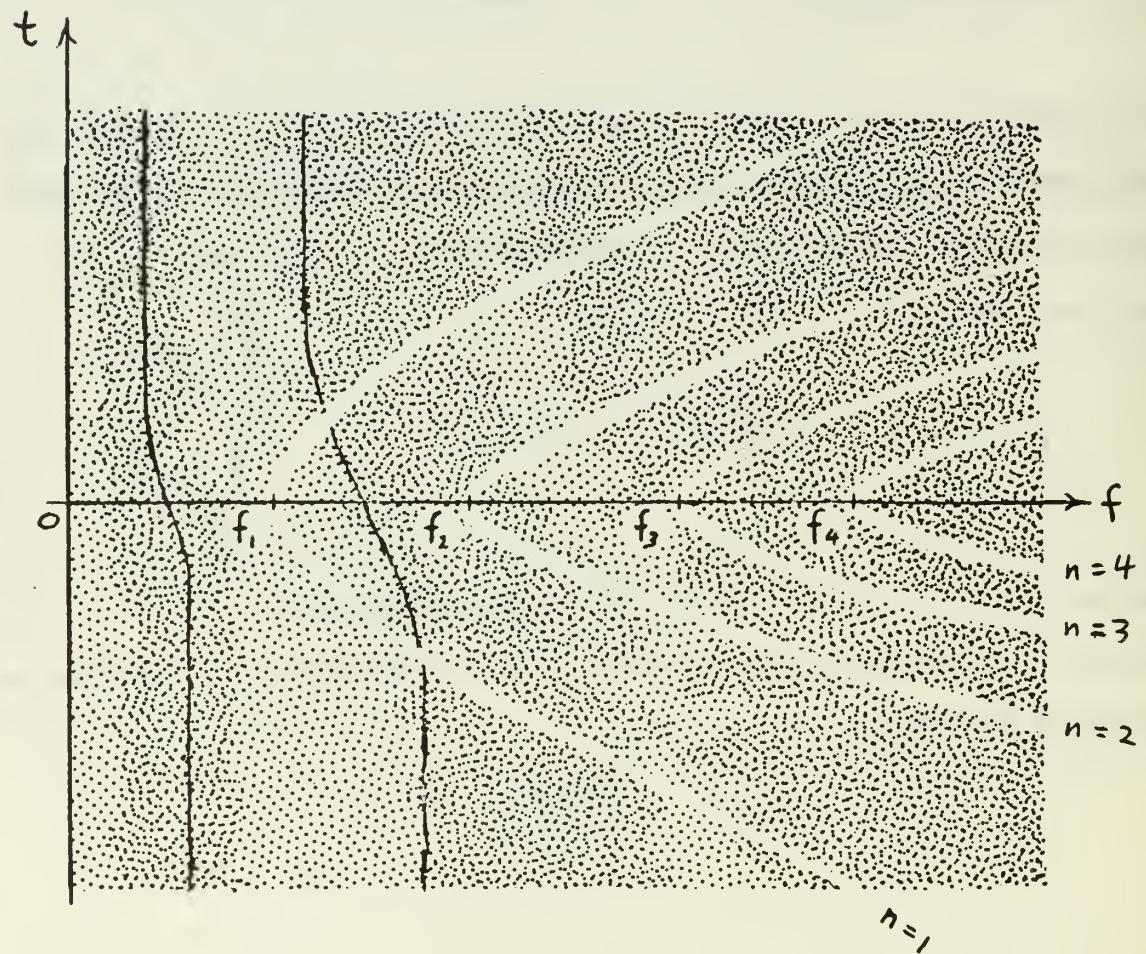
Our equation can be rewritten in standard form

$$\frac{f^2}{a^2} - \frac{t^2}{b^2} = 1$$

where $\quad a = \dfrac{n\,c\,s}{2\,h\,d}\quad$ and $\quad b = s/v$

This is the equation of a hyperbola
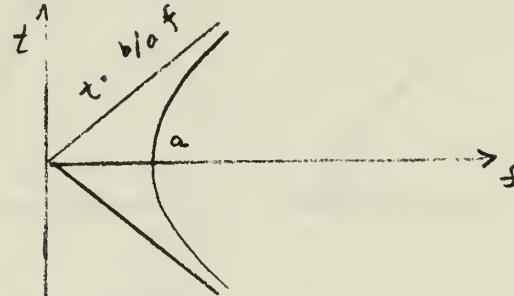
where a is the value of f at $t = 0$ and t/a is the slope of the asymptote.

If v is calculated from the measured Doppler shifts of the discrete lines, *
the depth of the source is found from

$$d = n\left(\frac{c\,v}{2\,h}\right)\frac{b}{a}$$

and the distance of closest approach from

$$s = \frac{1}{n}\left(\frac{2\,h\,d}{c}\right)a$$

Of course, for day-to-day use, nomograms can be developed to simplify the
calculations.

* It will be shown later that

$$\Delta f = 2\,\frac{v}{c}\,f$$

where Δf is the __total__ frequency change of a line.
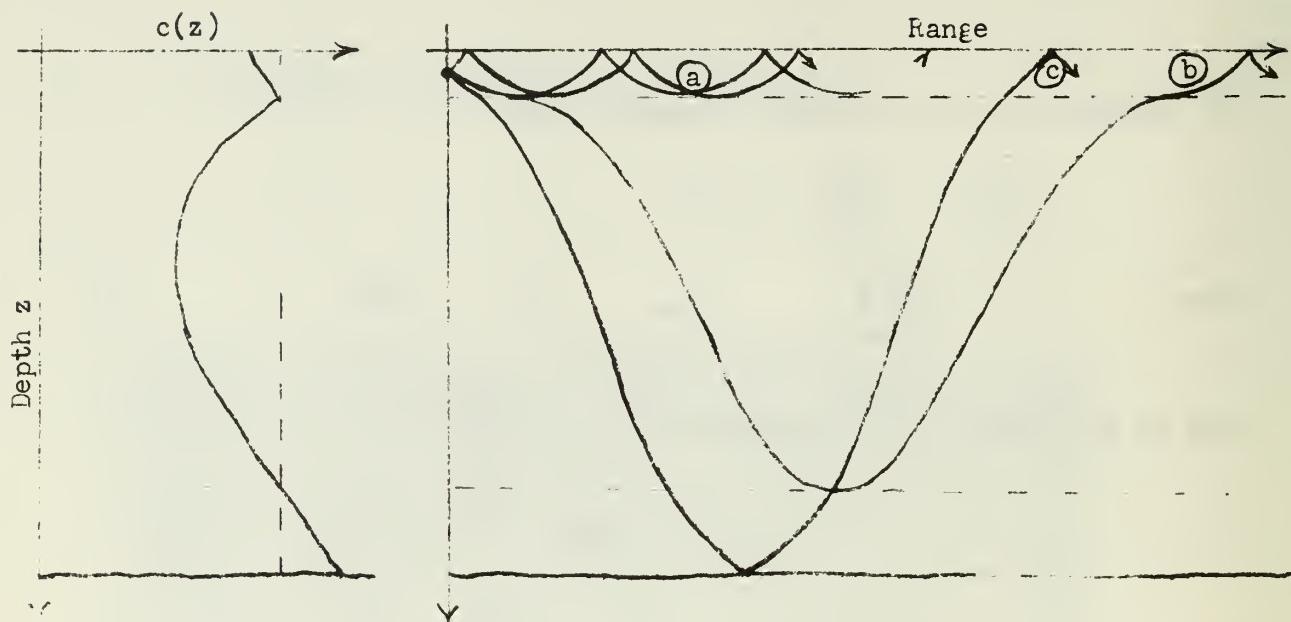
(a) <u>Surface Duct</u>. The isothermal water at the ocean surface forms
the mixed layer in which sound can be trapped. The continual
reflection at the rough surface, refraction at the bottom of
the layer, and leakage provide relatively large energy losses
so that this path is useful for relatively short ranges.

(b) <u>Convergence Zone</u>. When the water is deep enough, the sound rays
which travel down past the SOFAR channel axis can be bent back
upward before they reach the bottom. These rays, when they
approach the surface again, tend to be brought together so that
the sound energy becomes more concentrated, and the transmission
loss is reduced by a significant amount. Under advantageous
conditions, these rays will be refocused in second, third, and
higher convergence zones.

(c) <u>Bottom Bounce</u>. The reflection of sound off the bottom and back
to the surface provides a propagation path filling in the ranges
between the greatest useful range of the mixed layer and the
innermost edge of the convergence zone. Unfortunately, the
unpredictable and usually large reflection loss that occurs at
the bottom causes the transmission loss along this path to be
rather large.
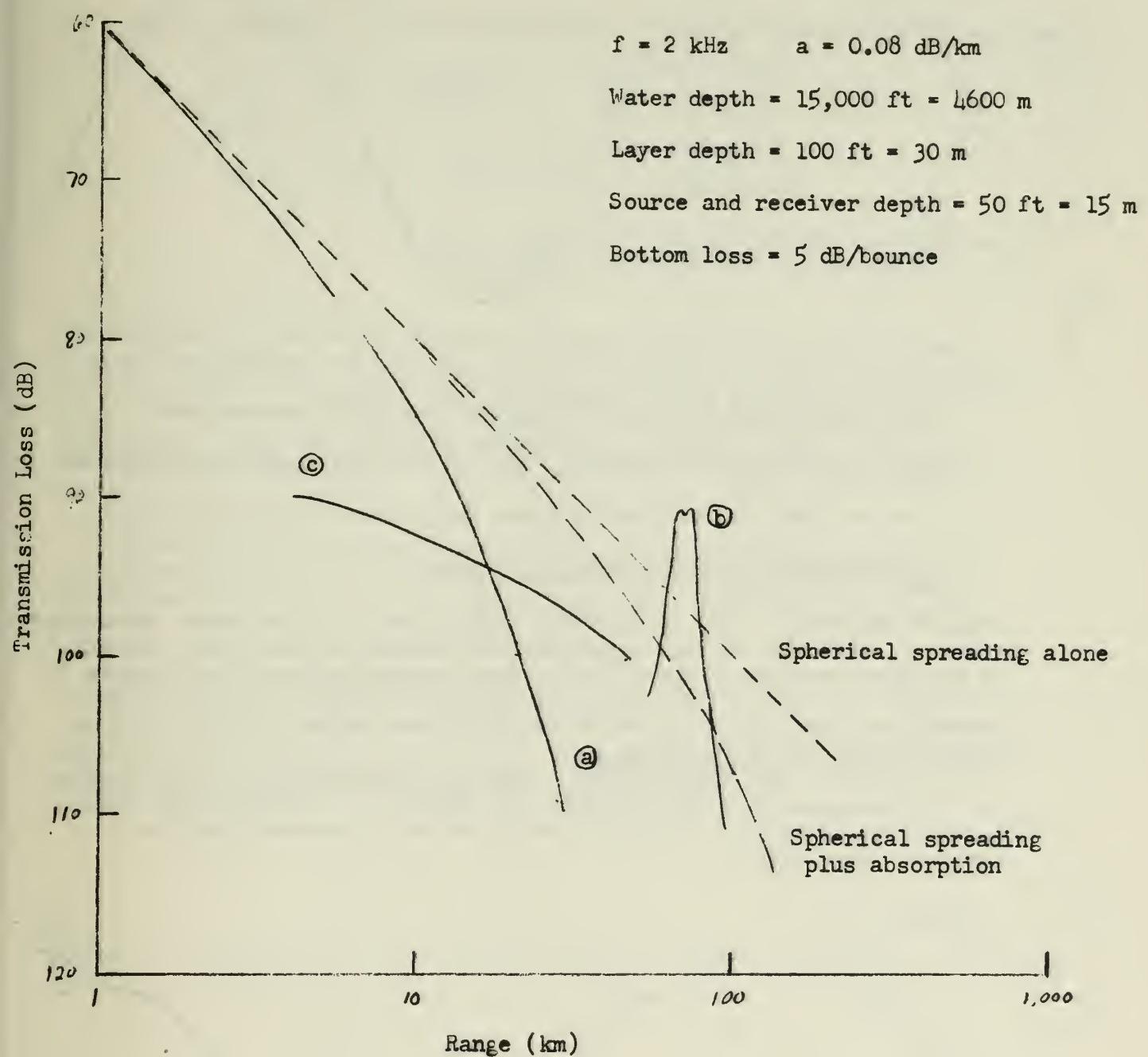
(a) Surface duct

(b) Convergence zone

(c) Bottom bounce

Ref. Urick, Principles of Underwater Sound for Engineers
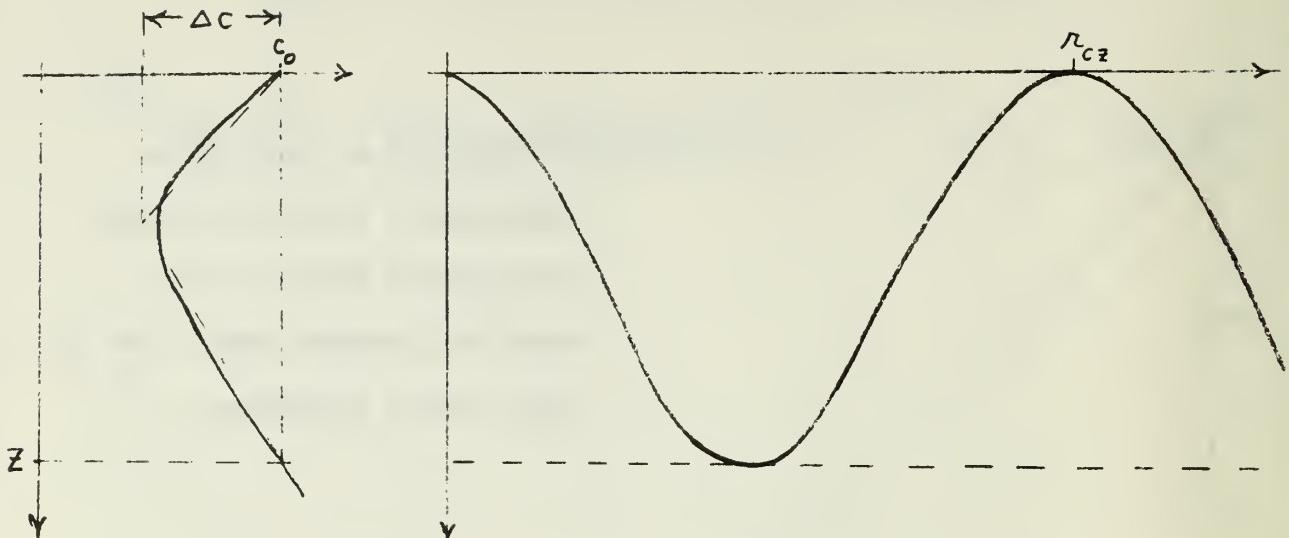
c_o = the maximum speed of sound above the SOFAR channel axis

Δc = the difference between c_o and the speed of sound at the elbow

Z = the vertical distance between depths for which $c(z) = c_o$

r_{CZ} = the range to the convergence zone

Snell's Law reveals that a ray which is horizontal in the upper ocean where $c(z) = c_o$ will also be horizontal at a distance Z deeper, and a ray which is directed downward where $c(z) = c_o$ must penetrate below the distance Z. To insure that enough rays are returned to the surface to focus at the convergence zone, the total depth of water must exceed Z or Z + D by an amount called the depth excess, d

If there is no surface layer, then c_o is associated with the surface and Z is measured from the surface. If there is a surface layer, then c_o is associated with the bottom of the layer and Z measured from the bottom of the layer.
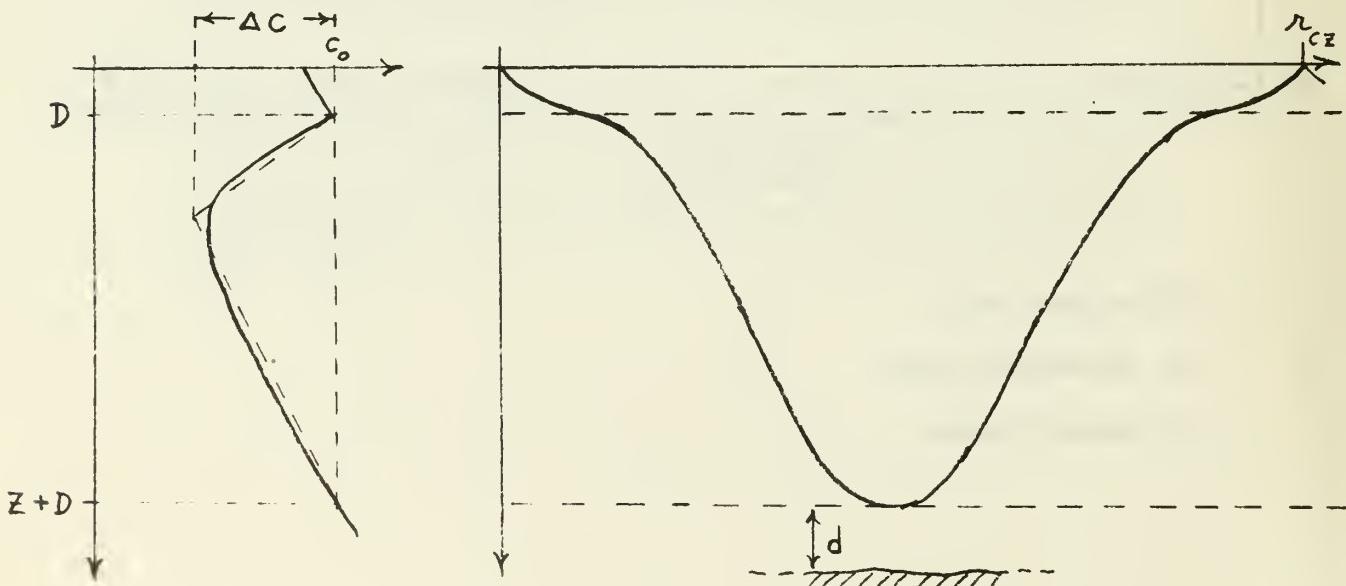
CONVERGENCE ZONE RANGE AND TRANSMISSION LOSS

A rough estimate of the range at which a convergence zone forms is given by

$$r_{CZ} \sim 2.8 \ Z \ \sqrt{c_o / \Delta c} \quad + \quad \text{Layer effect}$$

Layer effect = 0 if there is no surface duct

= r_s if a surface duct is present and both source and receiver are near the top of the layer.

The accuracy of the above expression depends on how well the constant-gradient approximations fit the true $c(z)$ profile. As might be expected, when the elbow of the curve is very sharp, and the gradients above and below the elbow are relatively constant, then the prediction will be rather good. The width of the convergence zone, over which the rays are appreciably focused, is normally about 10 per cent of the range to the zone.

Since the rays begin to spread out spherically as they leave the source and are attenuated only by the absorptive losses in the ocean, the transmission loss for convergence zone paths can be written in the form

$$TL = 20 \ \log \ r \quad + \quad ar \quad - \quad G$$

where G is the <u>convergence zone gain</u>. Typical values of G range between about 5 and 15 <u>dB</u> when a convergence zone is found.

Typical Convergence Zone Ranges

North Pacific

Surface Temperature*	Minimum depth of CZ operation+		Range to first CZ	
50° F	1270 ftm = 2320 m		47 kyd = 43 km	
55	1610	2940	52	47
60	1900	3480	56	51
65	2150	3930	60	55
70	2400	4400	64	57
75	2600	4760	66	60
80	2800	5130	69	63

Area	Surface Temperature*	Minimum depth for CZ operation+	Range to first CZ
N. Pacific	50° F	1270 ftm	47 kyd
N. Atlantic	50	1050	46
Norwegian Sea	50	1680	53
Mediterranean	67	800	33

* Assumes surface duct absent

+ Allows for a 200 ftm depth excess

Ref. Convergence Zone Range Slide Rule, Naval Underwater Systems Center

For angles of depression beyond about 15°, the radius of curvature of a ray is so large that it is a good approximation to assume straight-line propagation of the rays over the distances of interest.
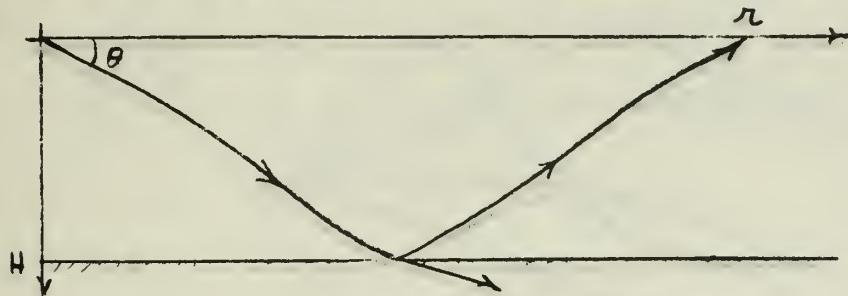
A ray bouncing off the bottom will return to the surface at a range

$$r = 2 H \tan \theta$$

Since the distance travelled by the ray is $r/(\cos \theta)$, and since the rays spread out spherically and are attenuated only by the absorptive losses of the ocean and the reflection at the bottom, an approximate transmission loss can be written as

$$TL = 20 \log \frac{r}{\cos \theta} + \frac{ar}{\cos \theta} + BL$$

where BL is the bottom loss in dB/bounce.

Depends on frequency and grazing angle

$$COT \; \ell \qquad \frac{r}{2H}$$
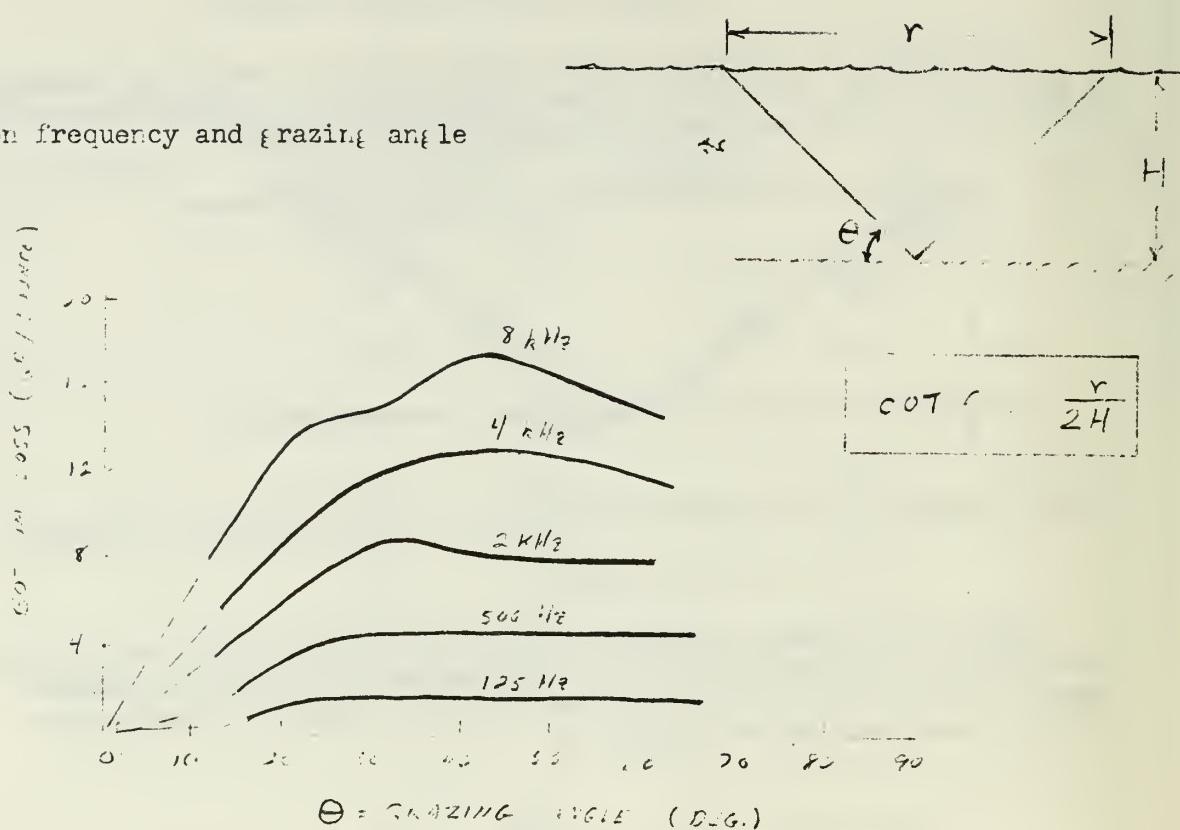
r/2H	1	1.19	1.43	1.73	2.14	2.75	3.73
θ	45°	40°	35°	30°	25°	20°	15°

Depends on bottom composition

    Example: At 24 kHz and 10° grazing angle

Mud	16 dB/bounce
Mud-sand	10
Sand-mud	6
Sand	4
Stony	4

Depends on surface roughness

    Bottom loss goes up as ratio of roughness-to-wavelength increases

Ref. Urick, Principles of Underwater Sound for Engineers
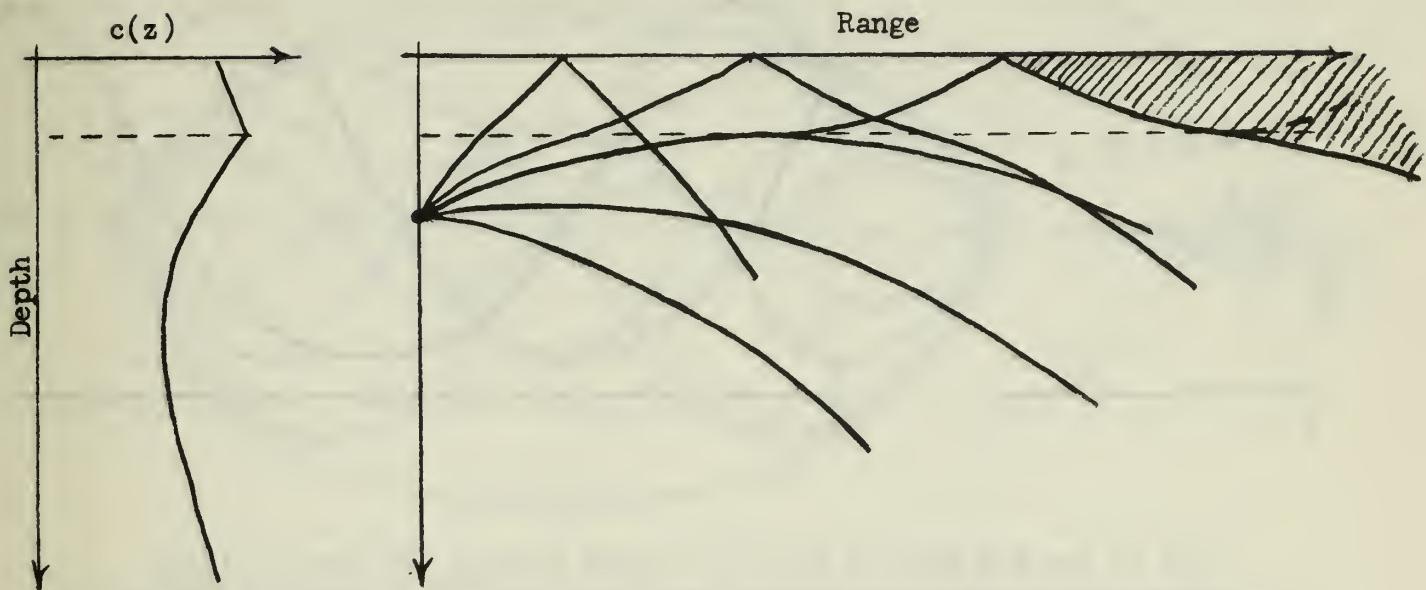
Reliable Acoustical Path (RAP).

If the target is shallow and the sensor deep, or vice versa, there exists the possibility of a guarenteed acoustical path between the two. This path is useful to ranges beyond those attainable by transmission in the surface duct. As the deeper of the two (sensor or target) is lowered further and further below the layer, the shadow zone indicated in the diagram moves out to larger ranges. If there is no mixed layer, then the leading edge of the shadow zone is formed by the downward-bending ray which just grazes the surface.

Because the angles for RAP are usually fairly large, the rays can be considered to travel approximately in straight lines between the target and sensor. For this kind of propagation, the rays spread out spherically and the only losses are those from the absorption of sound in sea water. An approximate transmission loss formula is then

$$TL = 20 \log \frac{r}{\cos \theta} + \frac{ar}{\cos \theta}$$

where θ is the angle of elevation of the rays between target and sensor.

If the negative gradient (below the layer, if one exists) has magnitude g, then the range to the shadow zone at the surface is estimated by

$$1.4 \sqrt{c_o(z-D)/g} + \tfrac{1}{2} r_s$$

where $(z - D)$ is the depth below the bottom of the layer. If there is no layer, then $D = 0$ and there is no skip distance, so $r_s = 0$.
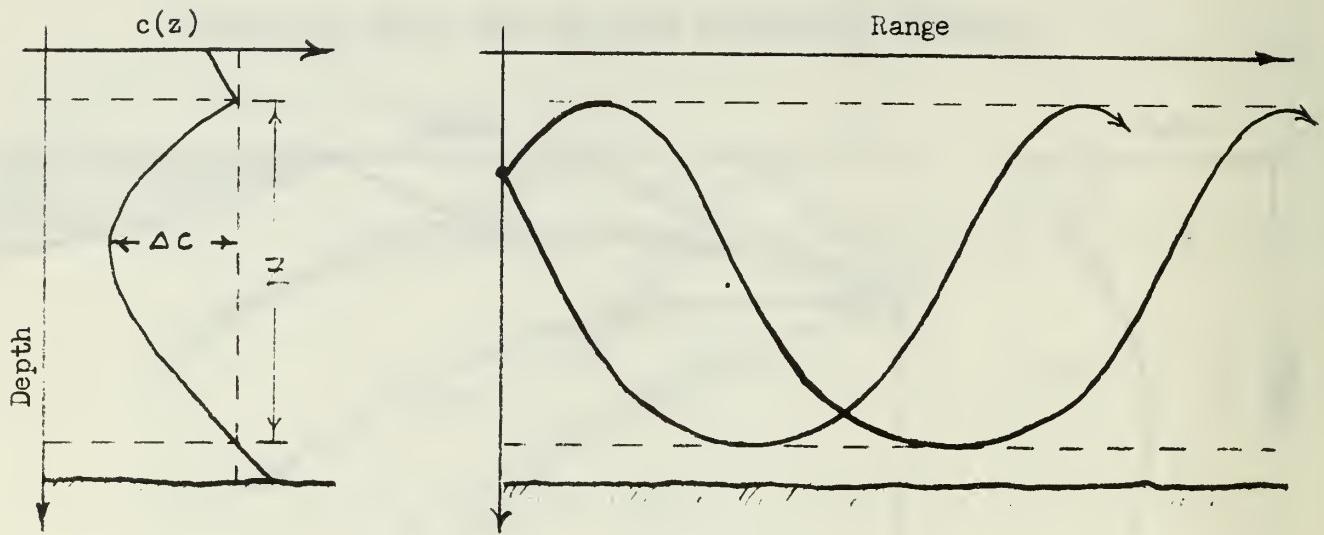
Just as for the surface duct, the sound spreads out more or less
spherically until the rays are bent back into the channel, after which
the sound spreads cylindrically. Thus, beyond some transition range r_t'
we should have a spreading term which goes as 10 log r. Those rays
whose angles of inclination with respect to the SOFAR channel axis are
large enough that they reflect from the ocean surface or bottom will
suffer reflection losses and scattering losses and eventually be so
weak compared to the trapped rays that they can be neglected. Thus,
for propagation in the channel, there are no losses except absorption,
so that the transmission loss has the simple form

$$TL = 10 \log r + 10 \log r_t' + ar \quad \text{for ranges beyond } r_t'.$$

The transition range depends on how deep the sound source is in the channel
and the vertical extent of the channel. For sources placed reasonably
down in the channel, the transition range is less than the convergence
zone range r_{cz}. Notice that the rays cycle up and down in the channel,
accomplishing one cycle over an interval very similar to the convergence
zone range.

$$r_t \sim \frac{\overline{z}}{2 \sin \theta_{max}}$$

TIME STRETCHING IN THE SOFAR CHANNEL
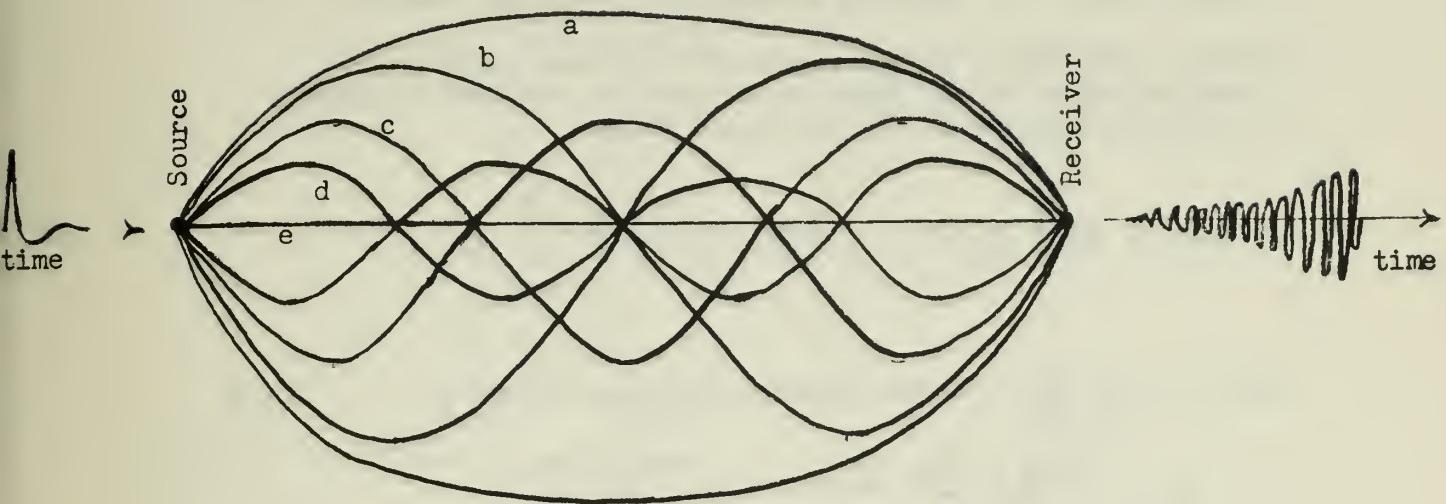
An interesting fact often observed in the SOFAR channel is that transient signals are "stretched" in time as they propagate to large ranges. This arises because of the variation in the speed of sound over the vertical extent of the channel. For the SOFAR channels commonly encountered in the northern hemisphere, the speed of sound increases sufficiently rapidly with distance above or below the channel axis that sound traveling over those rays which swing over large vertical distances, like a and b in the above sketch, reaches the receiver at times earlier than sound traveling over the more direct paths c, d, and e. Thus, the sound that travels straight down the axis of the channel arrives at the receiver last. This leads to the characteristic blossoming of the received signal from an explosive charge, the sound first being very weak, and then growing in volume until there is a sudden cutoff of sound as the last signals come over the slowest paths.

The exact details of these effects depend greatly on the character of the speed of sound profile: If the sound does not increase sufficiently rapidly above and below the axis, for example, the time stretching may not occur, or may even be reversed, with the sound traveling straight down the axis arriving first. Additionally, the exact location of the source and receiver with respect to the channel axis is very important with respect to the envelope of the received signal and the amount of time stretching encountered. For the typical profiles in the northern hemisphere, the increased duration Δt of the received signal over that sent can be estimated by

$$\frac{\Delta t}{t} = \frac{1}{3}\frac{\Delta c}{c}$$

if source and receiver are on the channel axis, and less if they are not. The quantity t is the total time of flight between source and receiver and Δc is the maximum variation in the speed of sound between the upper and lower boundaries of the channel indicated in the previous page. Typically, this leads to about 9 seconds of stretching for each 1000 nautical miles the signal has travelled.

Another approach to the propagation of sound in the ocean is Normal Mode Theory. If the mathematical difficulties of this approach can be overcome, it yields more exact values for the transmission loss than can be obtained using Ray Theory.

Normal Mode Theory yields a sum of individual solutions (each called a normal mode) which must be combined to obtain a complete expression for the sound field.

$$p = p_1 + p_2 + \cdots + p_n + \cdots = \sum_{n=1}^{N} p_n$$

Each one of these normal modes has its own behavior,

$$p_n = \underbrace{Z_n(z)}\ \underbrace{\frac{A_n}{\sqrt{r}} \sin\left[2\pi f\left(t - \frac{r}{c_n}\right)\right]}$$

This part looks just like a cylindrical wave traveling outward with its amplitude falling off like $1/\sqrt{r}$. Notice however that it has its own special speed c_n.

This part is an amplitude factor which changes with depth. Notice that each normal mode has its own special depth-dependent amplitude factor.

These special factors Z_n are called "normalized eigenfunctions". They are found by solving the wave equation and specifying the behavior of the pressure at the ocean surface and bottom. The solution for each Z_n also gives a required value for c_n. Both the depth dependence of Z_n and the value of c_n depend on the frequency of the sound waves.

This kind of a solution can be obtained whenever the speed of sound profile $c(z)$ is shaped so that the sound can be trapped in a channel. The trapping of sound in the mixed layer is one such example, and the propagation of sound to long ranges in the SOFAR channel is another.

$c(z)$

z

$Z_1(z)$ $Z_2(z)$ $Z_3(z)$ $Z_6(z)$

. . .   . . .

z
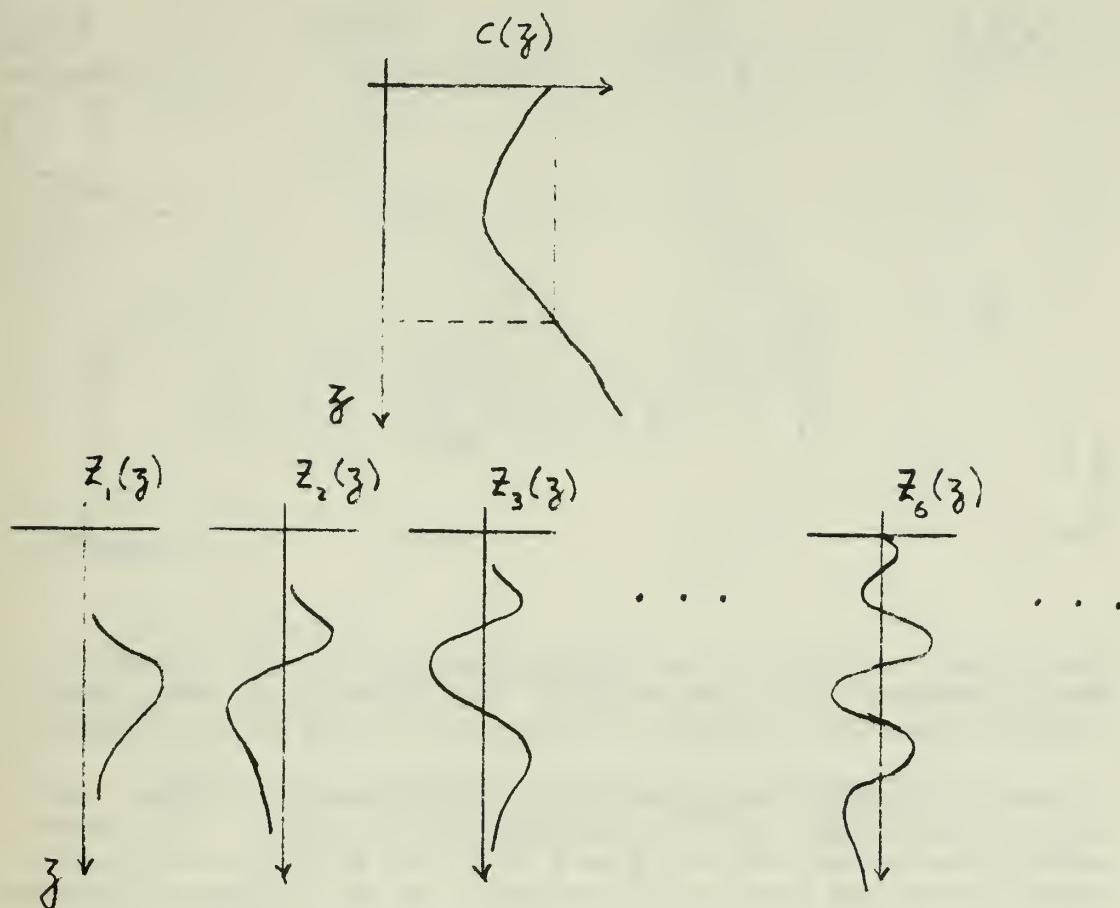
The above sketches represent the normalized eigenfunctions of the first three and sixth normal modes for sound trapped in the SOFAR channel. These sketches show that the greatest part of the pressure field is found in the channel, but the long "tails" which extent to depths far below the channel axis show that there is sound ensonification at depths greater than those predicted by drawing ray paths.
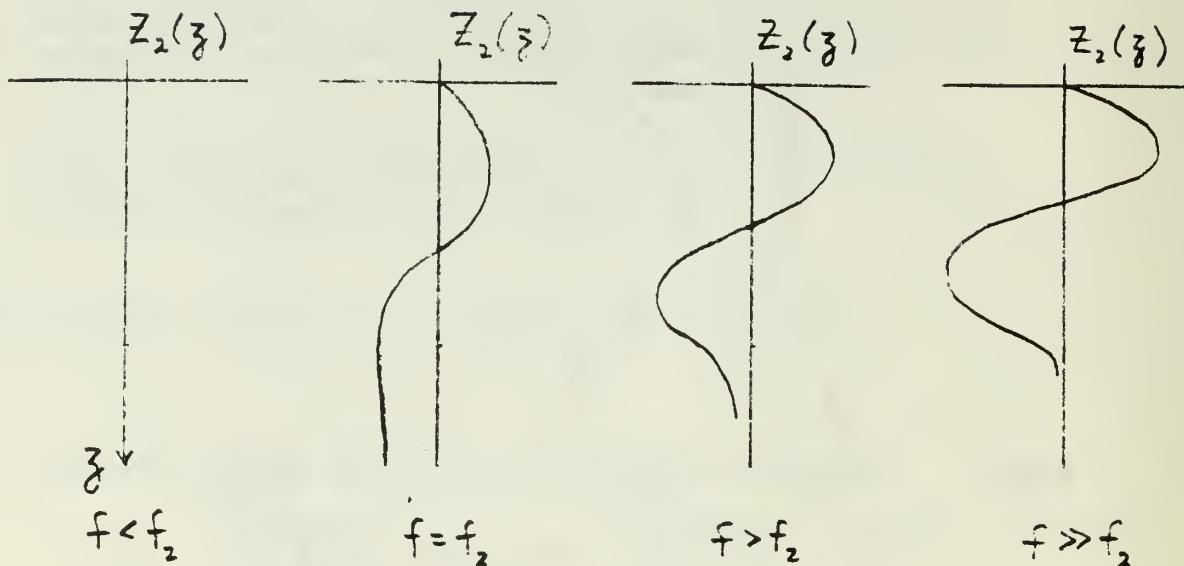
Each normal mode has its own <u>cutoff frequency</u> f_n. If the sound has a frequency below that of cutoff for a particular mode, then that mode cannot be excited to carry sound energy in the channel.

If the frequency of the sound is slowly increased with time, the above sketches show how a normal mode can be excited. Just above cutoff, the normal mode has a very long tail which goes to great depths beyond the edges of the channel. As the frequency increases further above cutoff, the tail gets shorter and shorter, until at frequencies well above f_n there is very little sound existing · outside the channel.

This means that for a specified frequency, sound can be trapped and sent to large distances only in those normal modes whose cutoff frequencies are less than the frequency of the sound. Normal modes with higher cutoff frequencies will have longer tails than those with lower cutoff frequencies, and those whose cutoff frequencies exceed the frequency of the sound cannot be excited at all. The larger the tail extending out of the channel, the more energy can be lost from the sound in the channel. This means that higher modes (those with higher cutoff frequencies) will attenuate faster and be less significant at great distances. This effect is sometimes called <u>mode stripping</u>.

These energy losses for each mode can be handled by specifying an absorption coefficient α_n for each normal mode. Then, we have

$$P_n = Z_n(z) \frac{A_n}{\sqrt{r}} \sin\left[2\pi f\left(t - \frac{r}{c_n}\right)\right] e^{-\alpha_n r}$$

where α_n increases with increasing n.

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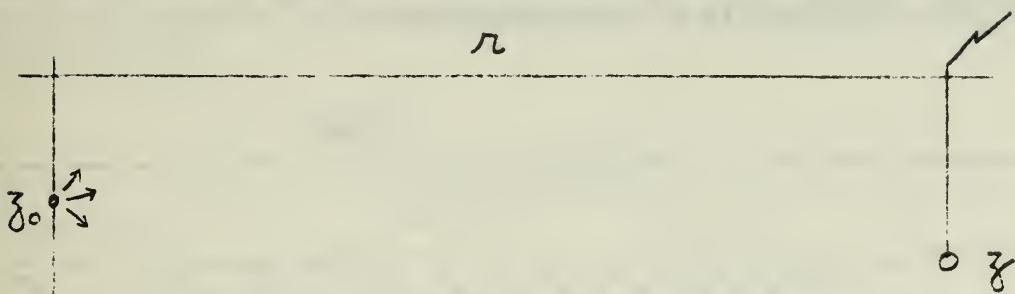
$$F_n = \sqrt{\frac{c_n}{f}} \; \frac{Z_n(\zeta_0)}{\sqrt{r}} \; Z_n(\zeta) \; e^{-\alpha_n r}$$

If <u>coherence</u>

$$P = \left| P_1 + P_2 + \cdots \right|$$

$$TL = -20 \log \left| \sum_n F_n \sin\left(\frac{2\pi f}{c_n} r\right) \right|$$

If <u>incoherence</u>

$$P = \sqrt{P_1^2 + P_2^2 + \cdots}$$

$$TL = -20 \log \sqrt{\sum_n F_n^2}$$

COHERENCE	INCOHERENCE
low frequency	high frequency
short ranges	long ranges
homogeneous water	inhomogeneous water
smooth boundaries	rough boundaries

RAYS, NORMAL MODES, AND REALITY

Normal mode theory is a complicated mathematical formalism which does not lend itself to simple intuitive interpretation. However, it is an exact theory which avoids certain approximations inherent in ray theory. It is useful in describing situations for which ray theory fails. An example of one such situation is illustrated below.
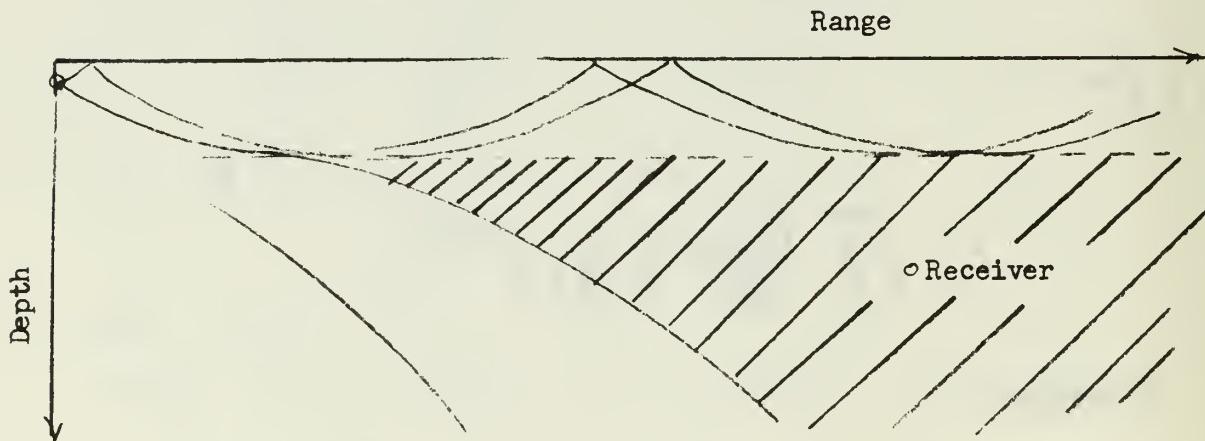
The source is in the layer and the receiver is below the layer. Ray theory predicts a shadow zone at large enough ranges. While there can be some scattering from the ocean surface into the shadow zone, this will be small for low frequencies and low sea states. The surface interference pattern at short ranges shows that the sea was quite smooth (the observed sea state was about 1 or 2).
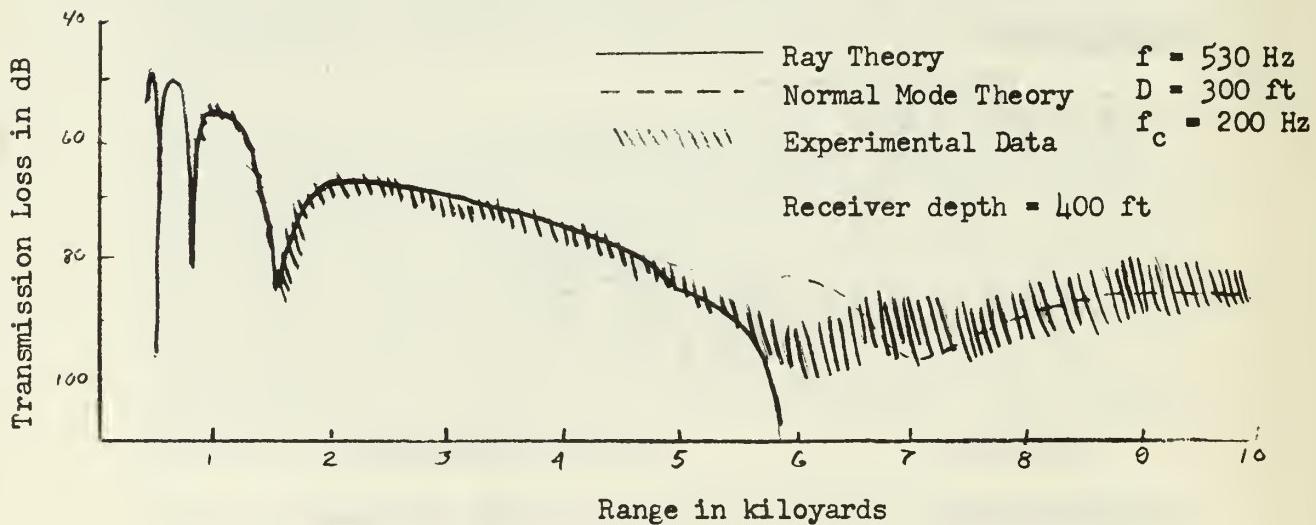
Notice that the experimental data show significant ensonification of the shadow zone. This cannot be predicted by the ray theory, and the scattering of sound from the surface is not strong enough to account for it.

Reference: Pederson and Gordon, Jour. Acoust. Soc. Am. 37, 105 (1965)

SUMMARY OF TRANSMISSION LOSS EXAMPLES

For both source and receiver in the surface layer, and at ranges greater than the transition range, the transmission loss can be estimated by

$$TL = 10 \log r_t + 10 \log r + ar + br/r_s.$$

For a convergence zone detection, we can write

$$TL = 20 \log r + ar - G.$$

If the propagation path is bottom bounce, and the angle of depression of the sound beam in that mode is sufficiently large, then the simple geometrical model gives

$$TL = 20 \log \frac{r}{\cos \theta} + \frac{ar}{\cos \theta} + BL.$$

For propagation between a deep point and a shallow point over the reliable acoustical path, an approximate transmission loss formula is

$$TL = 20 \log \frac{r}{\cos \theta} + \frac{ar}{\cos \theta}.$$

Finally, for the propagation of sound trapped in the SOFAR channel, if the range is greater than the transition range r_t', the transmission loss can be estimated by

$$TL = 10 \log r + 10 \log r_t' + ar.$$

Now, let us no longer restrict ourselves to tonals (monofrequency signals). Because we really don't deal with only a single frequency, we have to find out how signals of different frequencies combine. Consider the combination of a large number of monofrequency signals:
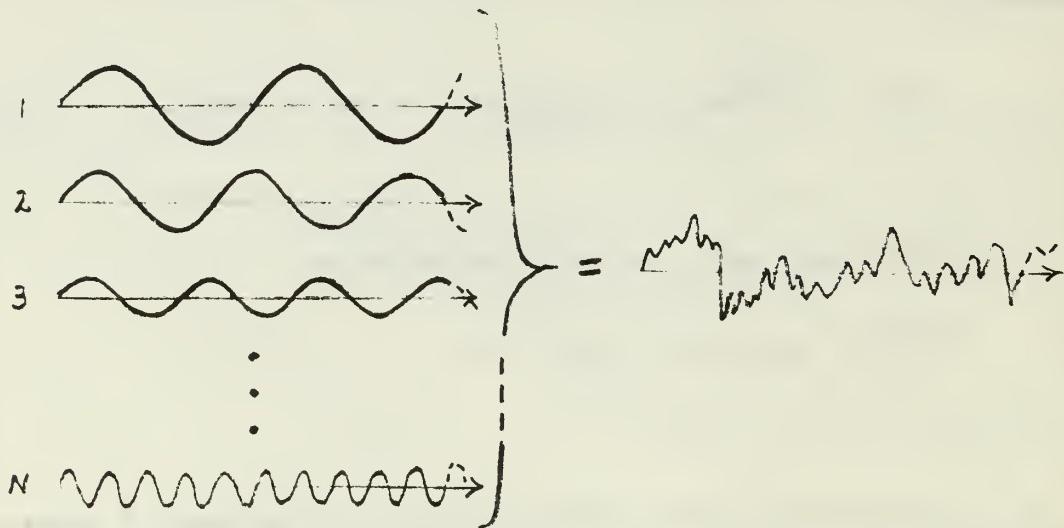
The combination of many signals, each a pure tonal, will result in a total signal which does not appear to have definite amplitude or period. However, the composite signal is an acoustical wave, and the total acoustical energy it possesses is the sum of the acoustical energy of each of the individual tonals. Thus, if E_T is the total energy of the composite signal, we have

$$E_T = E_1 + E_2 + \cdot \quad \cdot \quad \cdot \quad = \sum_{i=1}^{N} E_i$$

where E_1 is the energy of signal 1, E_2 that of signal 2, and so on.

For the complicated signals encountered in real life, which have large variations from moment to moment in their amplitude and shape, the frequencies of the individual tonals which combine to form them are extremely closely spaced. Indeed, it is often more convenient to talk about the energy contained in the collection of tonals which lie between some specific frequency and another frequency which is only infinitisimally different. If ΔE is the total energy of all the tonals lying within the incremental bandwidth Δw, then we can define the quantity ε as the average <u>energy density</u> over the incremental bandwidth Δw by

$$\varepsilon = \frac{\Delta E}{\Delta w} \quad .$$

Notice that the energy density has the units of energy/Hz. If we analyze some complicated signal, therefore, we can describe it in terms of the <u>energy density</u> it contains in each incremental interval of frequency over the entire range of frequencies which comprise it.

An example of the analysis of some complex signal into the energy densities found in each frequency interval is suggested in the following curve:
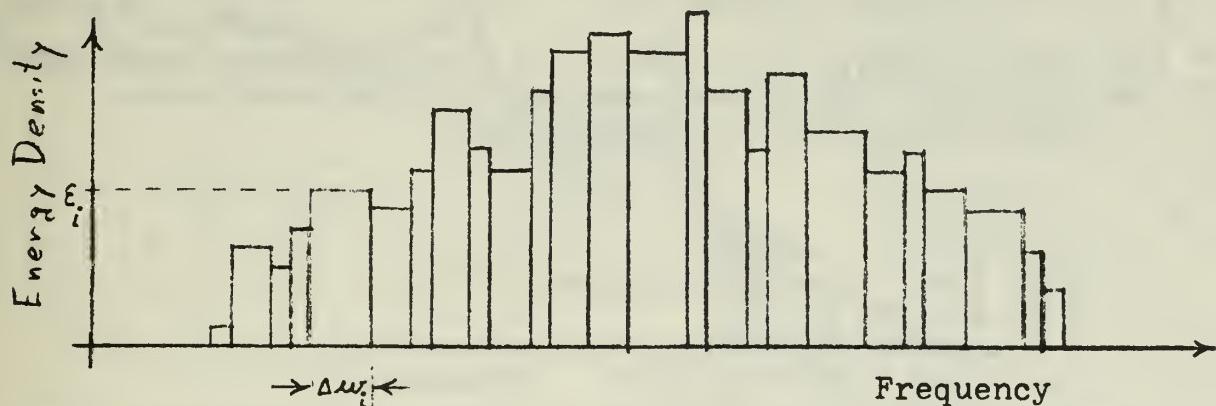
The total energy of the signal is given by

$$E_T = \varepsilon_1 \Delta w_1 + \varepsilon_2 \Delta w_2 + \cdots = \sum_i \varepsilon_i \Delta w_i$$

which is the area under the curve of energy density vs frequency.

If all of the Δw_i are chosen to be 1 Hz wide, then the graph gives the energy to be found in each 1 Hz interval, and the curve is called the <u>energy spectrum</u> of the signal.

It is more conventional in acoustics to describe the signal not in terms of the energy spectrum, but rather in terms of the <u>pressure spectrum level</u> PSL, which we shall define here as

$$PSL = 20 \, \log \frac{\rho \cdot 1 Hz}{P_{ref}} \qquad dB \; re \; P_{ref}$$

where the quantity ρ , the effective pressure per Hz, generates the associated energy found in the 1 Hz bandwidth. For example, if we receive a certain acoustical signal, pass it through a filter with a 1 Hz bandwidth, and obtain the effective pressure amplitude of the resultant filtered signal, say 100 μPa, then the value of ρ for that 1 Hz bandwidth is 100 μPa/1 Hz = 100 μPa/Hz. A representative curve of the pressure spectrum level vs frequency is presented below:
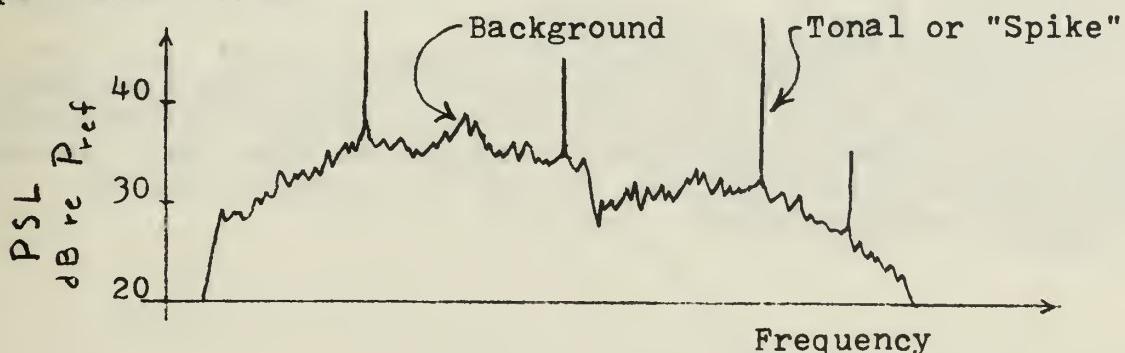
AVERAGED ENERGY SPECTRUM AND AVERAGED PSL

Very often, what is presented as the energy spectrum or the
pressure spectrum level is not that which would be measured with
a 1 Hz filter, but is rather calculated from measurements made
with a filter of larger bandwidth.

The overall energy to be found in some arbitrary bandwidth
w is the area under the energy spectrum vs frequency curve
over the appropriate range of frequencies:
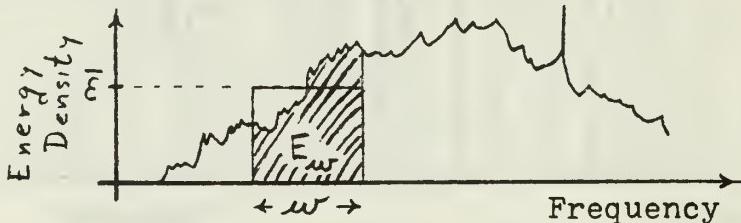
This gives the total energy E_w found within this bandwidth, and
does not say how ϵ is distributed over w. We can therefore write
the <u>averaged energy spectrum</u> $\bar{\epsilon}$ as $\bar{\epsilon} = E_w/w$. A plot of $\bar{\epsilon}$ vs
frequency would give a smoothed curve, lacking the detail of the
true curve, but sufficient to calculate the total energy present
over bandwidths comparable to or larger than the bandwidths selected
to obtain $\bar{\epsilon}$.

Analogously, it often happens that what is presented as a
pressure spectrum level is in reality a smoothed curve, the result
of plotting data obtained with filters having bandwidths appreciably
larger than 1 Hz:
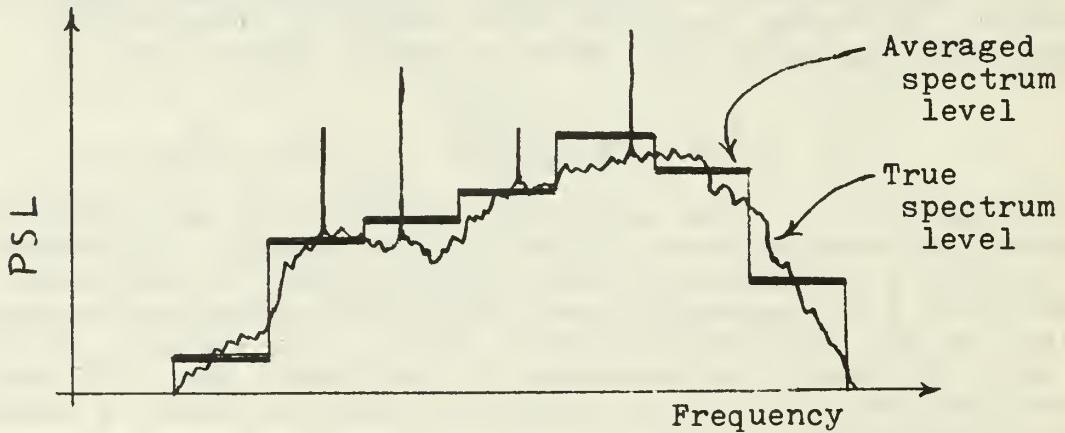
Use of too broad a bandwidth in calculating averaged spectrum
levels will wash out details. The spikes in the true spectrum
level in the above example are lost to view because the bandwidth
over which the averaged levels are taken is so wide that the energy
from the spike is small compared to all the energy from the
background in each bandwidth.
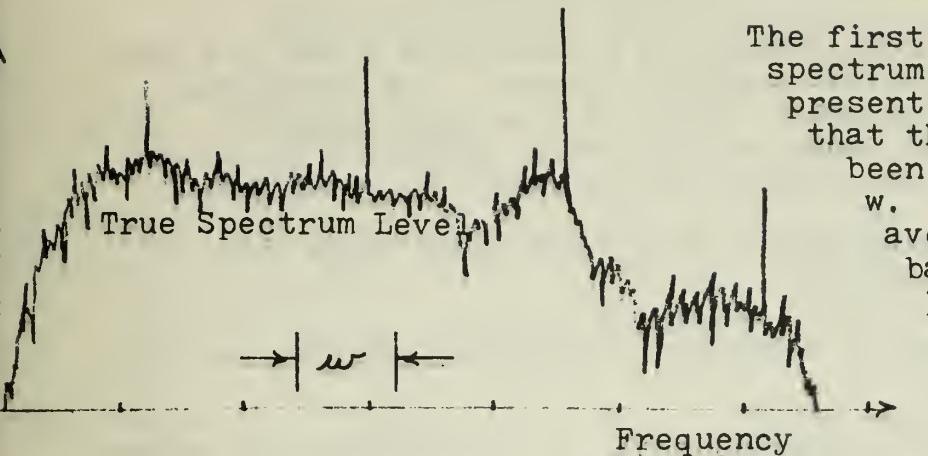
The first graph presents the true spectrum level and the other three present the same spectrum, except that the spectrum levels have been averaged over the bandwidth w. Graph 2 presents the averaged spectrum level in a bar graph format. Graphs 3 and 4 connect the data with either straight-line segments or a smooth curve. Notice that Graph 4 gives no information relating to the bandwidth of the filter used to obtain the averaged spectrum levels; this would have to be indicated separately, and is not likely to be done. Graph 3 is somewhat better, but the use of the straight-line segments seduces the eye into assuming that the true spectrum level closely follows the straight-line segments. Graph 2 is the least subject to being misinterpreted: The width of the individual bars gives the bandwidth of the filter quite visually, and each bar strongly emphasises by its flat top that the value of the PSL appropriate to that frequency interval is the averaged value.

Because of the fact that the averaged spectrum level smooths out and erases detail which would otherwise be found in the true spectrum level, it is very important that the extent of averaging over frequency be well-indicated. This is accomplished quite easily and unambiguously by the format of either Graph 2 or Graph 3. Further, Graph 2 clearly indicates the averaging of data by the flat-topped nature of the curve, whereas Graph 3 can tend to mislead the eye in a casual examination.

True Spectrum Level

Frequency

w

Frequency

Frequency
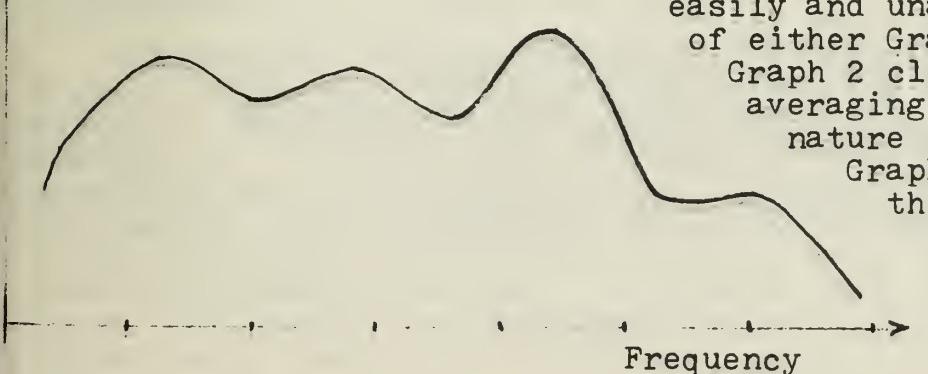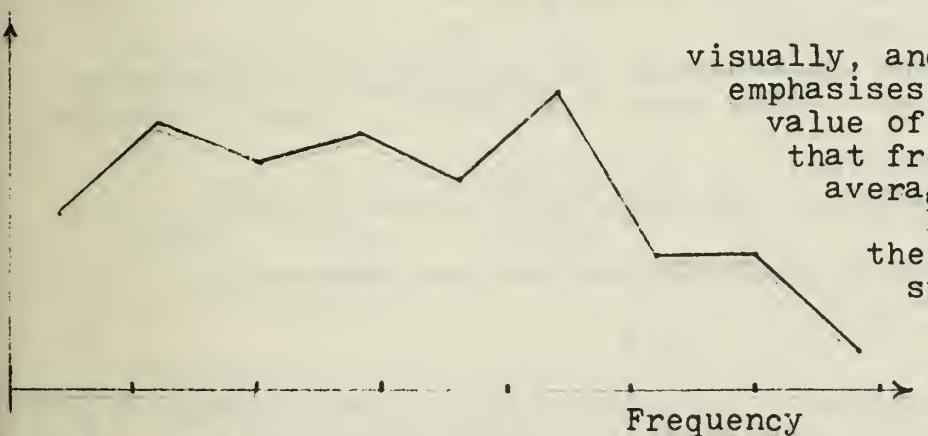
Frequency

SPECTRUM LEVEL AND BAND LEVEL

Recall that the total energy in a bandwidth w was the area under the energy density vs frequency curve. The analogous quantity in terms of pressure levels is the <u>band level</u> BL. This is the expression in dB terms of the total effective pressure generating the acoustical energy in the specific bandwidth.

NOTICE THAT THE BAND LEVEL IS <u>NOT</u> THE AREA UNDER THE SPL vs FREQUENCY CURVE.

This is because the log of a product is the sum of the logs of the individual factors.

Consider a plot of energy density ε vs frequency as shown:
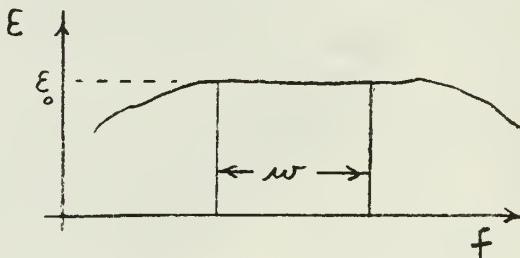
It is clear that the total energy E in the bandwidth w is given by

$$E = \varepsilon_o w$$

If we express this in log form, we have

$$\log \frac{E}{E_{ref}} = \log \frac{\varepsilon_o \cdot 1 Hz}{E_{ref}} + \log \frac{w}{1 Hz} \; .$$

It turns out in acoustics that taking $10 \cdot \log(E/E_{ref})$ is the same as taking $20 \cdot \log(P/P_{ref})$. This means that the above equation is the same as

$$20 \log \frac{P}{P_{ref}} = 20 \log \frac{P \cdot 1 Hz}{P_{ref}} + 10 \log \frac{w}{1 Hz}$$

or, treating the quantity 1 Hz as understood but suppressed,

$$BL = PSL + 10 \log w$$

In general this expression is exact <u>only</u> if the spectrum level is constant over the bandwidth w. However, it is reasonably accurate if the true spectrum level does not fluctuate by more than a couple of dB over the range of w.

Notice that this formula can be turned around: If a band level BL has been experimentally obtained by measuring the acoustical energy over some bandwidth w, then the averaged spectrum level over this bandwidth can be found from

$$PSL(averaged) = BL - 10 \log w$$

COMPARISON OF A SPECTRUM LEVEL AND A BAND LEVEL CURVE
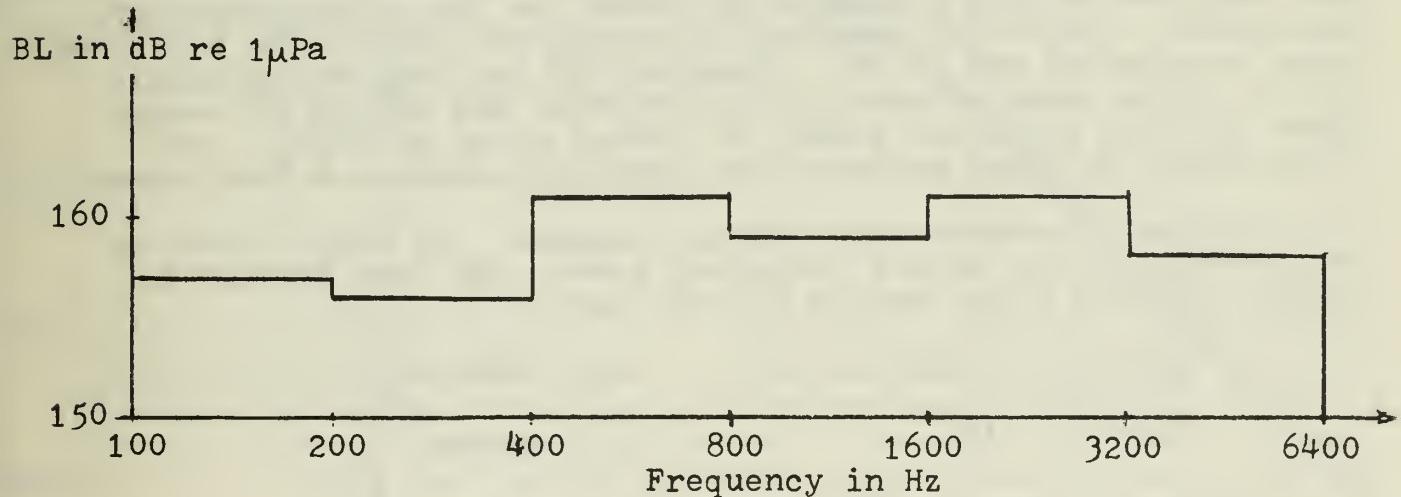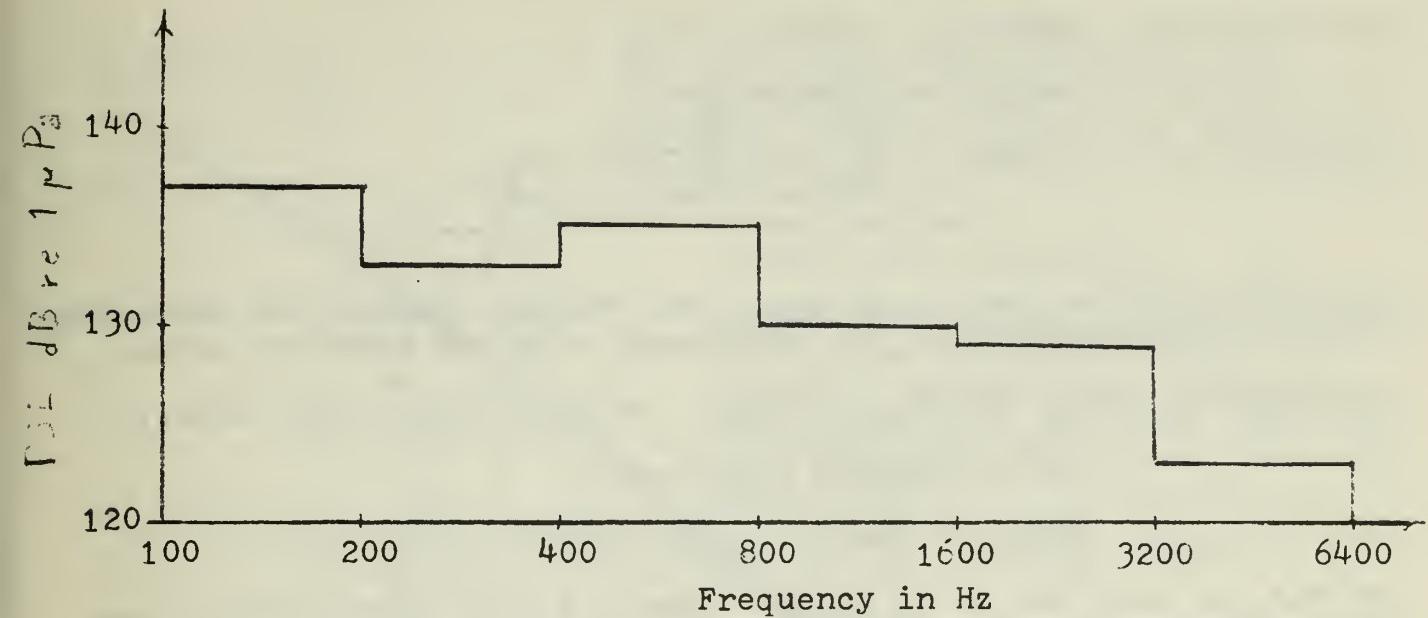
Because of the (10 log w) term which must be added to the PSL to obtain the appropriate BL for a certain frequency interval, it is seen that the shape of the BL curve is such that the BL is stronger at higher frequencies. This is because the BL measures the total acoustical energy in some frequency interval, whereas the PSL measures the energy per unit Hz.

Both kinds of curves have their advantages and disadvantages. A curve of the true spectrum level gives the maximum possible amount of information, but requires considerable mathematical analysis if it is desired to obtain the band level within some specified frequency interval from the spectrum level. A curve of averaged spectrum level is not quite so difficult (timeconsuming) to use, and band levels from it can be found more easily, but often important details are lost. A curve of band level is quite direct to use, as long as the bandwidth agrees with the particular receiving system being used, but it also has smoothed out much detail.

A SAMPLE CALCULATION

As a simple example, let us obtain the overall band level for the PSL curve presented below:
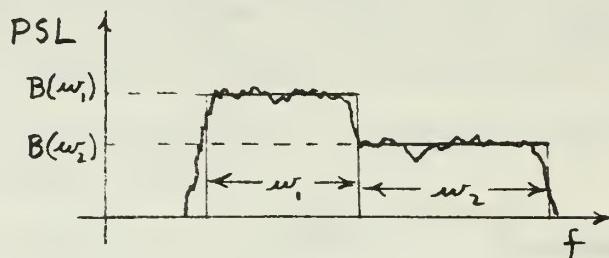
From the steplike nature of the curve, we can specify two bandwidths w_1 and w_2 as shown, with the associated averaged spectrum levels.

For each of these, we can calculate the appropriate band level:

$$BL(w_1) = B(w_1) + 10 \log w_1$$

$$BL(w_2) = B(w_2) + 10 \log w_2$$

So far, we have had to do no more than a little curve-fitting and application of formulas which have been presented previously. Now, however, a new problem has appeared. We must combine the two band levels into a single bandlevel for the total frequency interval $w_1 + w_2$. The combined BL is _not_ the sum of the individual ones, because we are in dB: Energies add but logs of energies do not. What must be done is to determine the amount of energy given by $BL(w_1)$ and the amount of energy given by $BL(w_2)$, and these energies added together and then expressed as a band level.

Let us illustrate this with an example. We shall first do the example in the method indicated above, and then present a simple nomogram which avoids all the work.

$$B(w_1) = 140 \, dB \, re \, 1\mu P_o \qquad w_1 = 1000 \, Hz$$

$$B(w_2) = 135 \qquad\qquad w_2 = 2000$$

$$BL(w_1) = 140 + 10 \log 1000 = 170 \, dB \, re \, 1\mu P_o$$

$$BL(w_2) = 135 + 10 \log 2000 = 168$$

$$E(w_1)/E_{ref} = antilog \frac{170}{10} = 10^{17}$$

$$E(w_2)/E_{ref} = antilog \frac{168}{10} = 10^{16.8}$$

$$E/E_{ref} = \frac{E(w_1) + E(w_2)}{E_{ref}} = 10^{17} + 10^{16.8}$$

$$BL(w_1 + w_2) = 10 \log \left(10^{17} + 10^{16.8} \right) = \underline{172.1 \, dB \, re \, 1\mu P_o}$$

NOMOGRAM FOR COMBINING LEVELS

This nomogram allows the combination of two different band levels (or two independent sound pressure levels) by graphical means. If the two levels to be combined are L_1 and L_2, the procedure is as follows:

Obtain the difference $L_1 - L_2$.

Locate this number on the horizontal axis, move straight up until the curve is reached, and then move horizontally to the left until the vertical axis is reached, and read that number.

Add that number to L_1.

$BL_1 - BL_2$

As an example of the application of this nomogram, recall the problem worked out on the previous page. The two band levels which were calculated there were $BL_1 = 170$ dB re 1μPa and $BL_2 = 168$ dB re 1μPa. The difference is +2 dB; finding this value, projecting upward, and reading to the left gives 2.1 dB, so that the combination of these two levels yields a total band level of 170 + 2.1 = 172.1 dB re 1μPa. Since the usual uncertainty in levels exceeds a few tenths of a dB, it is appropriate to round this combined level to 172 dB re 1μPa.
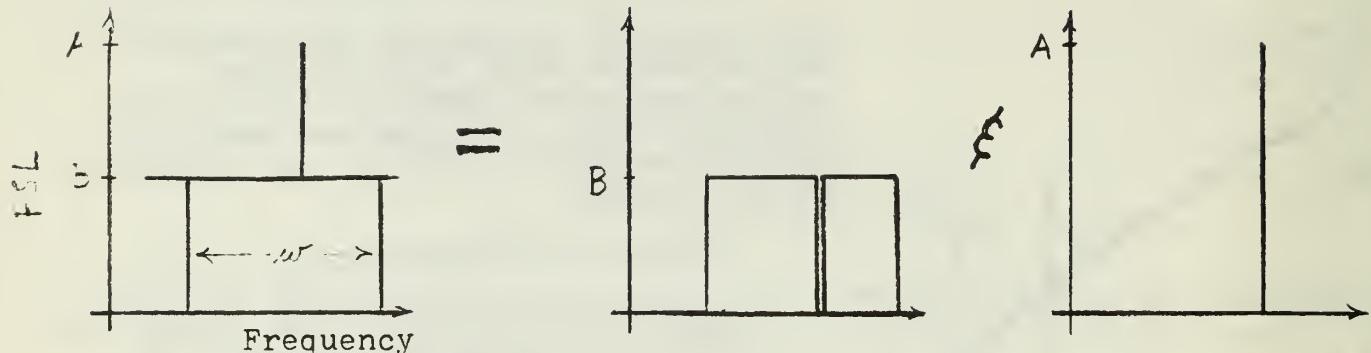
When there are strong tonals present, the band level cannot be expressed as B + 10 log w, because of the additional energy contributed by the spike(s). We must calculate the energy from the background spectrum (without the spikes) and add this to the energy of the monofrequency spikes which are also present:

$$BL_B = B + 10 \log(w-1) \doteq B + 10 \log w \quad \text{if } w \gg 1$$

$$BL_A = A + 10 \log 1 = A.$$

The total band level is found by combining BL_A and BL_B with the help of the nomogram.

Notice that if the tonal is relatively strong and the bandwidth narrow enough, the combined level will be due essentially to the tonal. As the bandwidth w increases, however, the influence of the (10 log w) term will become stronger, causing BL_B to increase. Eventually the contribution from the background will get sufficiently large that BL_B will be sufficiently large that the influence of BL_A will be lost.

$$EL \geq ML + DT$$

EL = <u>Echo Level</u> for active sonar or the received <u>Signal Level</u> for passive sonar. It is the level in dB of the desired signal when it reaches the receiver.

ML = <u>Masking Level</u>. It is the level in dB of the undesired signal at the receiver which is competing with the Echo Level.

DT = <u>Detection Threshold</u>. This is the number of dB by which the Echo Level must exceed the Masking Level to allow the receiving system to register a detection with a specified degree of confidence. This quantity involves the probability of detecting the target in the presence of noise and also the probability of giving a false alarm.

The SONAR equation states that signal at the receiver must equal or exceed background noise by an amount specified by the Detection Threshold if a specified degree of performance is to be achieved.

EL = Echo Level

ML = Masking Level

DT = Detection Threshold

SL = Source Level

TL = Transmission Loss

PSL = Pressure Spectrum Level

BL = Band Level

SSL = Source Spectrum Level

AG = Array Gain

NL = Noise Level

NSL = Noise Spectrum Level

DI = Directivity Index

w   = bandwidth

d   = detection index

wt = time-bandwidth product
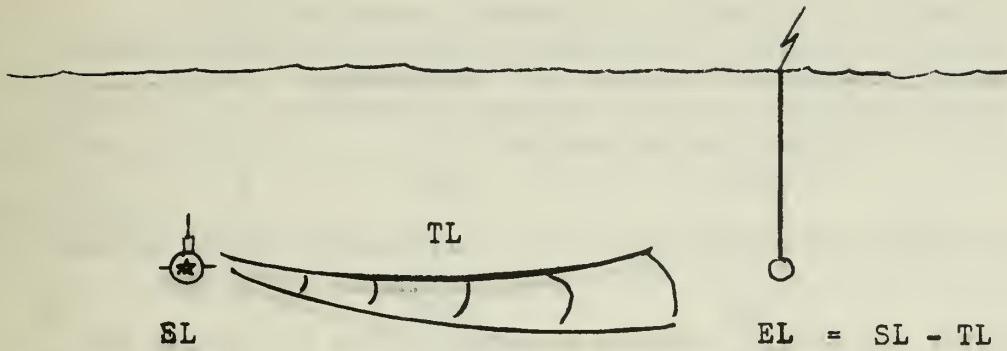
$$\boxed{SL \; - \; TL \; \geqslant \; ML \; + \; DT}$$

where SL = Source Level of the target .

TL = One-way transmission loss

Each term in the above equation will now be discussed separately.

$$\boxed{SL} - TL \geqslant ML + DT$$

SOURCE LEVEL - PASSIVE SONAR

I. Tonals (monfrequency components)

   Sources: Propulsion machinery (Main motors, reduction gear)
             Auxiliary machinery (generators, pumps, air-conditioners)
             Propeller-induced resonant hull excitation
             Hydrodynamically-induced resonant excitation of cavities,
                  plates, and appendages

   SL = Level of acoustic energy 1 meter from the acoustic center of the
         source.

         Measurements are usually made at a distance from the source and
         extrapolated back to 1 meter.

II. Broadband noise

   Sources: Cavitation at or near the propeller
             Radiated flow noise
             Caviation at struts and appendages

   SL = SSL + 10 log w

         where w is the bandwidth of the system and

         SSL = pressure spectrum level 1 meter from the acoustic center
                of the source.

                If the SSL is not essentially constant over w, the bandwidth
                must be subdivided into smaller segments, the BL of each
                found, and then all the BL's combined to give SL

TYPICAL SOURCE SPECTRUM LEVELS

Typical source spectrum levels for a conventional submarine on electric drive at periscope depth. SSL re 1μPa.

freq (Hz) Speed (knots)	100	1000	10,000
4	133	120	100
6	140	127	112
8	143	133	119
10	147	135	122

Adapted from Urick, Principles of Underwater Sound for Engineers

$$SL - TL \geqslant \boxed{ML} + DT$$

MASKING LEVEL
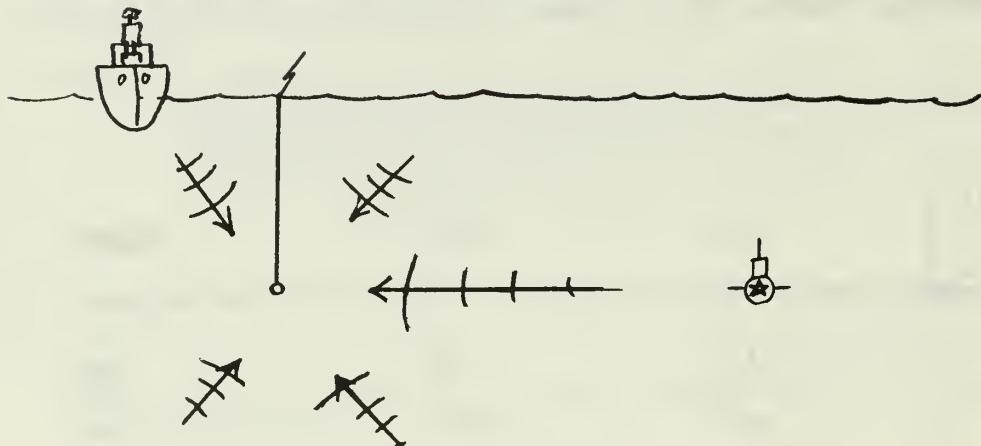
Unwanted noise may come in uniformily from all direction or it may arrive from a prefered direction.

$$\boxed{ML = NL - AG}$$

where

ML = masking level

NL = noise level in the environment

AG = array gain, the improvement (reduction) in dB against the noise field resulting from the array design.

$$ML = NL - AG$$

Noise Level

$$\boxed{NL = NSL + 10 \log w}$$

where

        NL = noise level, the acoustic energy in dB found in the bandwidth
                of the system ,

        NSL = noise spectrum level, pressure spectrum level of the unwanted
                noise,

and

        w = the bandwidth of the system in Hz.

Note: The same limitations apply in using this formula as were
        pointed out in the discussion of the broadband SL.

$$NL = \boxed{NSL} + 10 \log w$$

NOISE SPECTRUM LEVEL (Deep water)
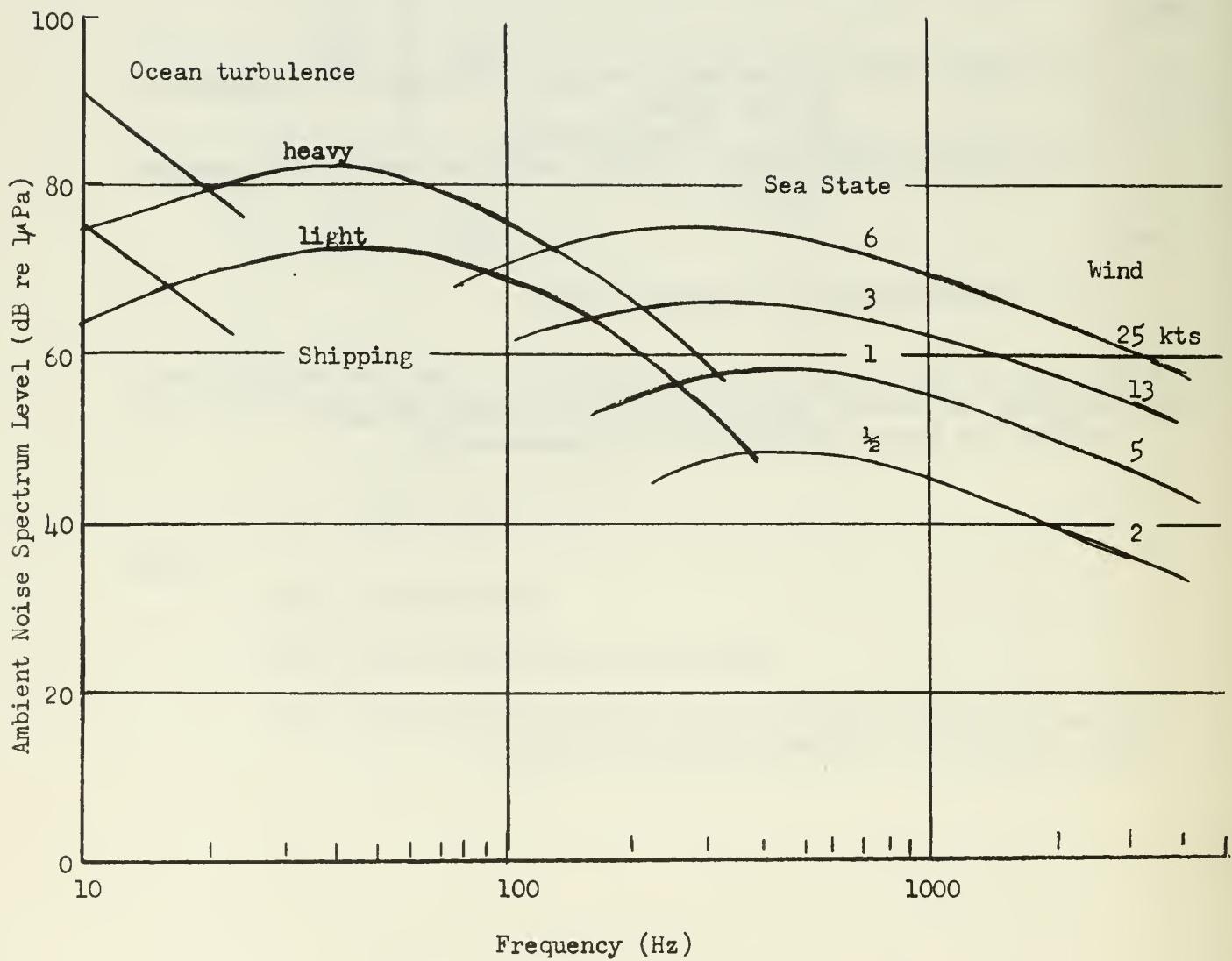
Adapted from: Wenz, Jour. Acoust. Soc. Am. 34 , 1936-1956 (1962)
            Perrone, ibid 46, 762-770 (1969)

$$NL = \overbrace{NSL} + 10 \log w$$

NOISE SPECTRUM LEVEL (Self-noise)

Sources of self-noise: Hydrodynamic noise due to flow past receiver
                         Acceleration of receiver
                         Machinery noise of own "ship"

Values usually presented as an "equivalent omnidirectional" noise spectrum level.

$$NL(\text{self-noise}) = NSL(\text{self-noise}) + 10 \log w$$

If significant self-noise is present, it must be combined with the ambient noise level to obtain the total noise level. Since these two sources of noise are independent, their energies add and the nomogram can be used.

$$ML = NL - \textcircled{AG}$$

ARRAY GAIN AND DIRECTIVITY INDEX

Array gain is the decrease in masking level from the noise level brought about by the design of the receiving array.

If the noise is isotropic (arriving uniformly from all directions) and the signal arrives from one direction only, then

$$AG = DI$$

where

DI = Directivity Index

= 10 log (the ratio of the noise energy sensed by an omnidirectional receiver to that sensed by the directional receiver)

In this case,

$$ML = NL - DI$$

Array gain can include both the design of the receiving array and also the signal processing. Signal processing such as correlation, by taking into account the different spatial or temporal properties of the signal and noise can result in an array gain different from the directivity index.

Add together,
get N times
the signal
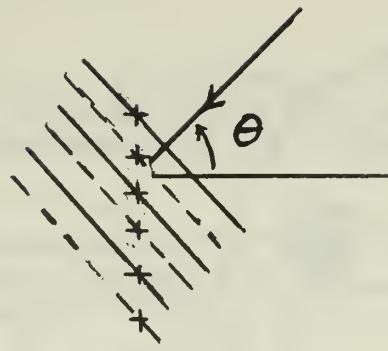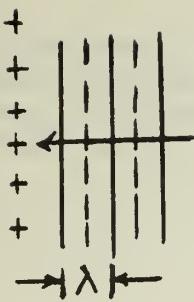
Pair by pair,
the signals
cancel

Resulting response
as a function of
angle:

P
P_{axial}
Major lobe
Side lobes
θ

Polar presentation:

weak
reception

strong
reception

$20 \log (P/P_{axial})$

90°
180°
-20 -10 0
$\theta°$
270°
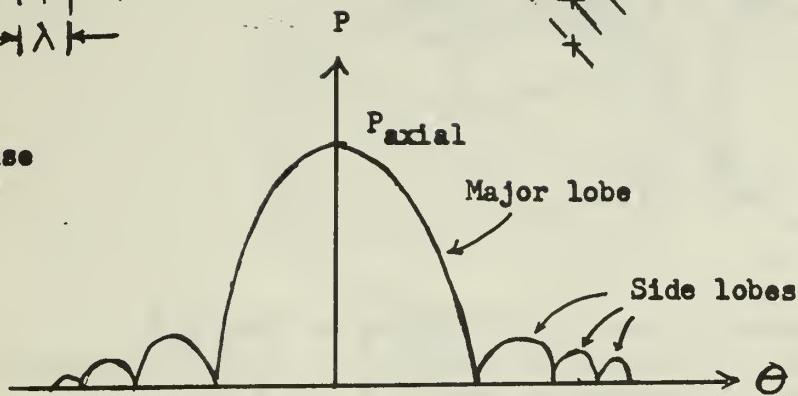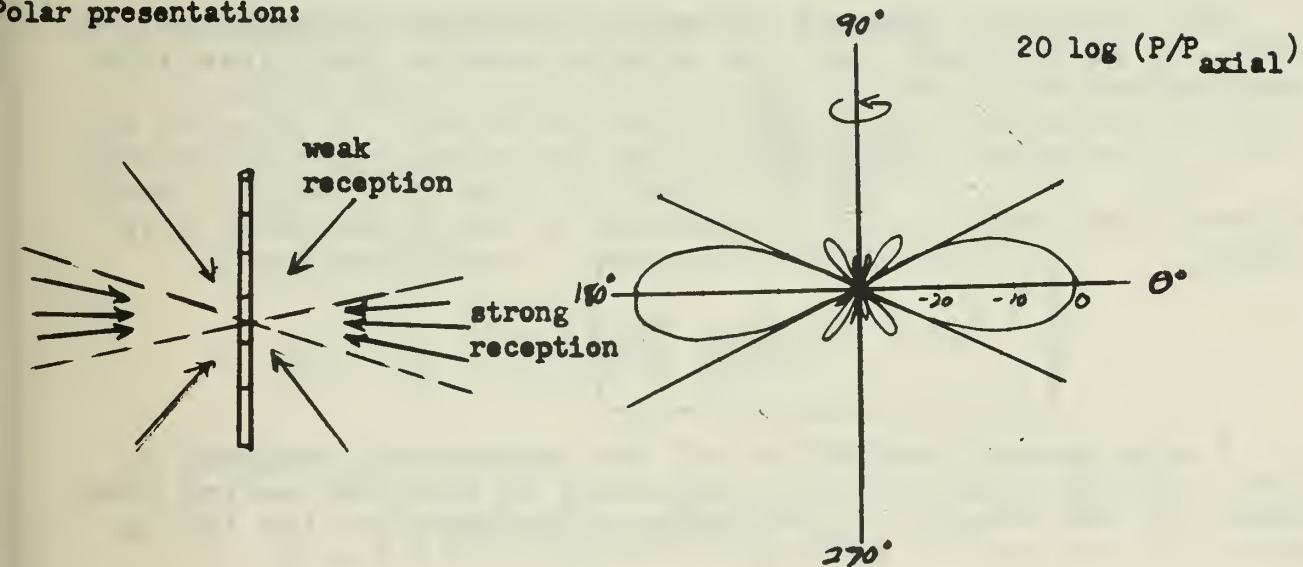
DI denotes how directional the receiver is.

DI = 0 dB Omnidirectional receiver

DI = 20 dB Receiving only 1% of the energy present in the noise field.

DI depends on size of the array compared to the wavelength. For example, for a long line array of length L much greater than λ

$$DI = 10 \log (2L/\lambda)$$

Generally, as frequency increases the receiver becomes more directive (DI increases) but, if the frequency is too high we may get more major lobes.

75

Sphere of unit radius

Illuminated patch on the unit sphere
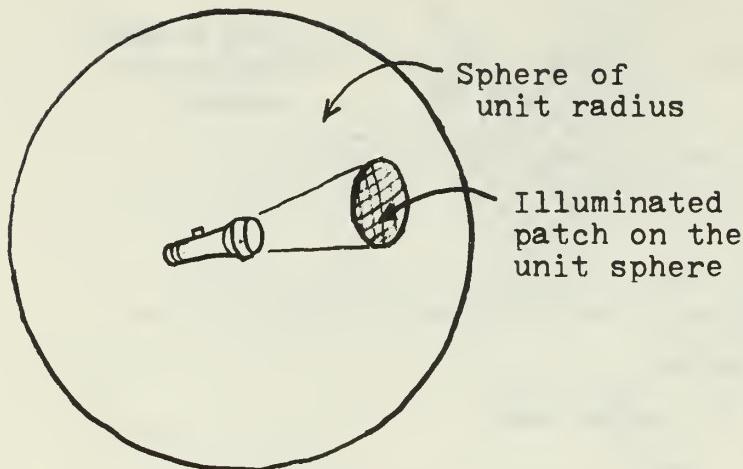
Surface area of the unit sphere $= 4\pi(1)^2$

Area of the illuminated patch $= \Omega(1)^2$

where Ω is the solid angle subtended by the patch.

The fractional area of the sphere which is illuminated is given by $\Omega/4\pi$ and, for this simple example, the directivity D is given by

$$D = \frac{4\pi}{\Omega} \cdot$$

The directivity index is the expression of the directivity D in dB terms,

$$\boxed{DI = 10 \log D} \cdot$$

A more general definition of the directivity is given as follows: If the pressure field generated by an array acting like a source (or the relative sensitivity of an array acting like a receiver) is written in the form

$$P(r,\theta,\emptyset) = P_{ax}(r) \, H(\theta,\emptyset)$$

where P_{ax} is the pressure on the acoustical axis of the array, then $H(\theta,\emptyset)$ is normalized to have a maximum value of unity on the acoustical axis, and to be no greater than unity for any other direction. The directivity can then be expressed as

$$\boxed{D = \frac{4\pi}{\displaystyle\int_{4\pi} H^2(\theta,\emptyset)\, d\Omega}} \cdot$$

If we have a line array with a single major lobe, and if the angular width of the major lobe is θ (in radians), then if the array is surrounded by a unit sphere, the illuminated patch appears as below
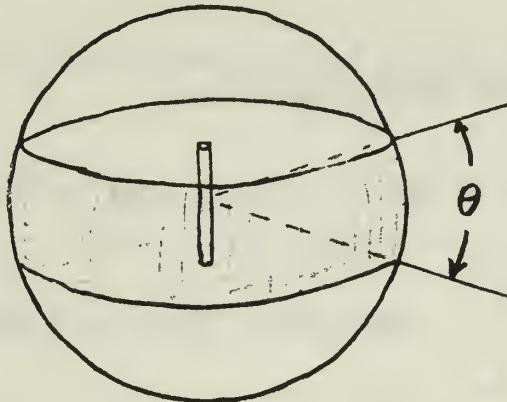
and the directivity can be crudely estimated to be

$$D \sim \frac{4\pi(1)^2}{2\pi\cdot(1)\cdot\theta\cdot(1)} = \frac{2}{\theta} \qquad (\ \theta \text{ in radians }\ )$$

where it must be understood that θ is small.

A little more careful estimation of the directivity from the geometry of the radiation (or reception) pattern leads to some relatively simple but only approximate estimates for the directivities of arrays of simple design,

Line-like arrays:

$$D \sim \frac{\pi}{2} \frac{\text{Array Length}}{\lambda} \gg 1$$

Piston-like arrays:

$$D \sim \left(\pi \frac{\text{Piston Diameter}}{\lambda} \right)^2 \gg 1$$

These formulas can be used as rough estimates if the arrays are highly directive (DI in excess of about 6 dB at the least), are designed to have only one major lobe, and are not strongly shaded or steered.

An array is shaded when different elements have different amplifications. This is done to alter the width of the major lobe and the relative strengths of the side lobes. Narrowing the major lobe will increase the strengths of the side lobes, and vice versa. An array is steered to point the major lobe in various directions with respect to the array. This is done by introducing time delays in the outputs of the elements.

PASSIVE SONAR EQUATIONS

Let's put it all together:

BROADBAND WITH NEGLIGIBLE TONALS

$$SSL + 10 \log w - TL \geqslant NSL + 10 \log w - AG + DT$$

$$\boxed{SSL - TL \geqslant NSL - AG + DT}$$

TONALS WITH NEGLIGIBLE CONTRIBUTION FROM BACKGROUND

$$\boxed{SL - TL \geqslant NSL + 10 \log w - AG + DT}$$

FIGURE OF MERIT

(FOM)

The passive SONAR equations can be rewritten by solving forTL:

$$TL \leq FOM$$

where

$$FOM = SL - (NSL + 10 \log w - AG + DT)$$

or

$$FOM = SSL - (NSL - AG + DT)$$

for tohal or broadband detection respectively.

Graphically:
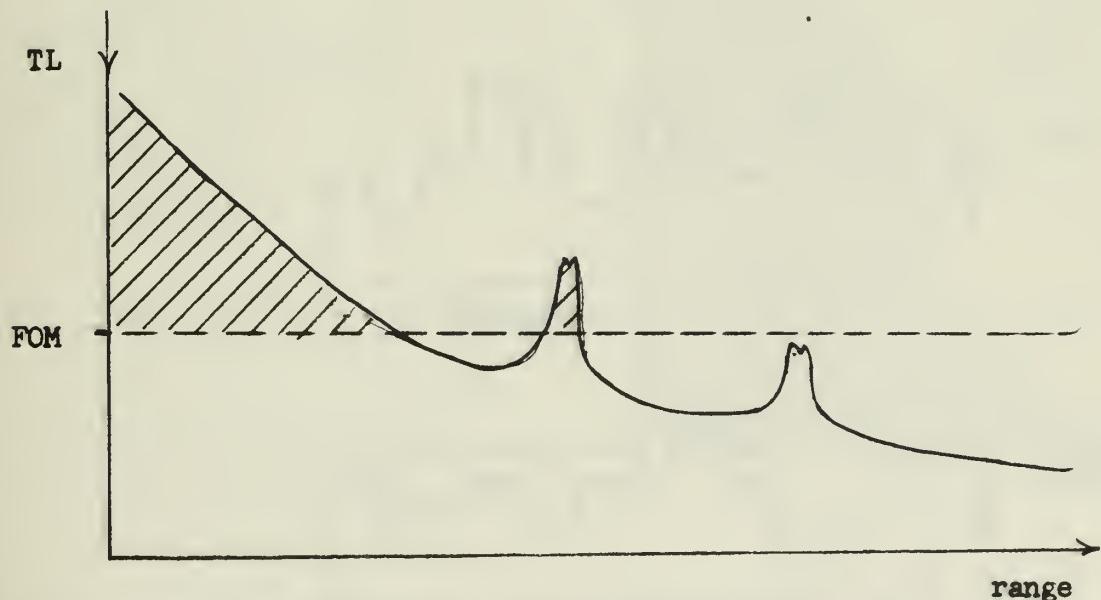
//////// = ranges for which detections are possible at conditions
equal to or better, than specified by DT.

To take advantage of a narrow bandwidth and still permit reception of a Doppler shifted signal, it is necessary to design a system that employs many parallel filters which, while individually narrow, cover the entire frequency range of interest.

The output of such a system might be presented in this manner:
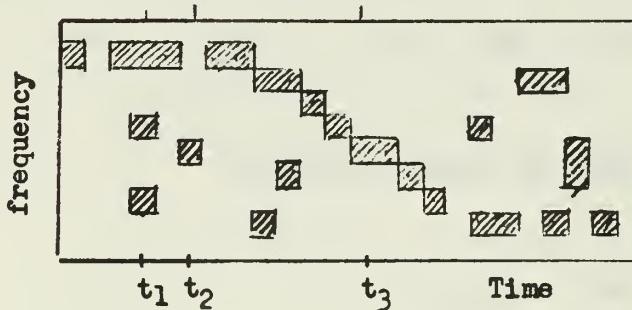
At any instant, the output of each of the individual filters might look like this:

At t_1:

frequency

At t_2:

frequency

At t_3:

frequency
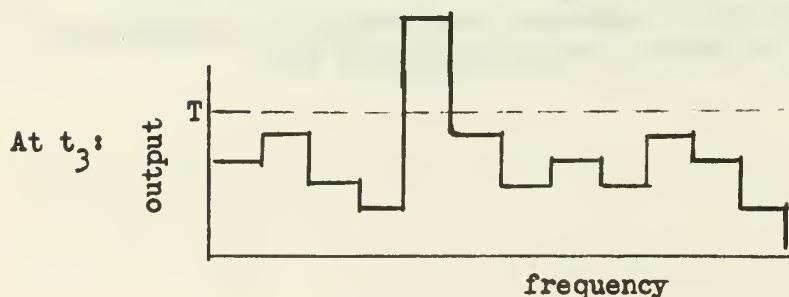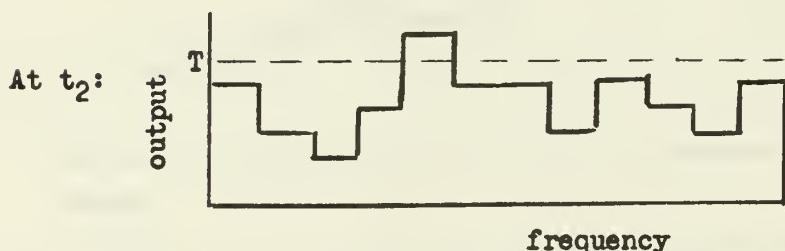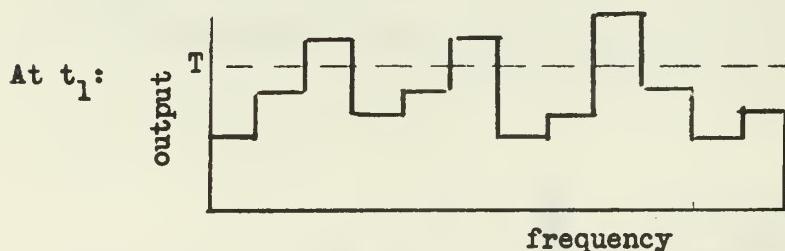
where T is the threshold setting.

$$SL - TL \leqslant ML + \boxed{DT}$$

DETECTION THRESHOLD

The detection process consists of designating a threshold which, when exceed, causes a detection to be recorded. If the signal is much stronger than the noise, it is clear that a threshold can be defined that will allow   valid signals to be recorded while   ' ignoring the noise. However, when the signal and noise are of comparable size, any threshold that will catch a reasonable number of valid signals will also record "detections" when a valid signal is absent.

The probablity of obtaining a certain amplitude of the noise alone as well as for the signal-and-noise is sketched below:
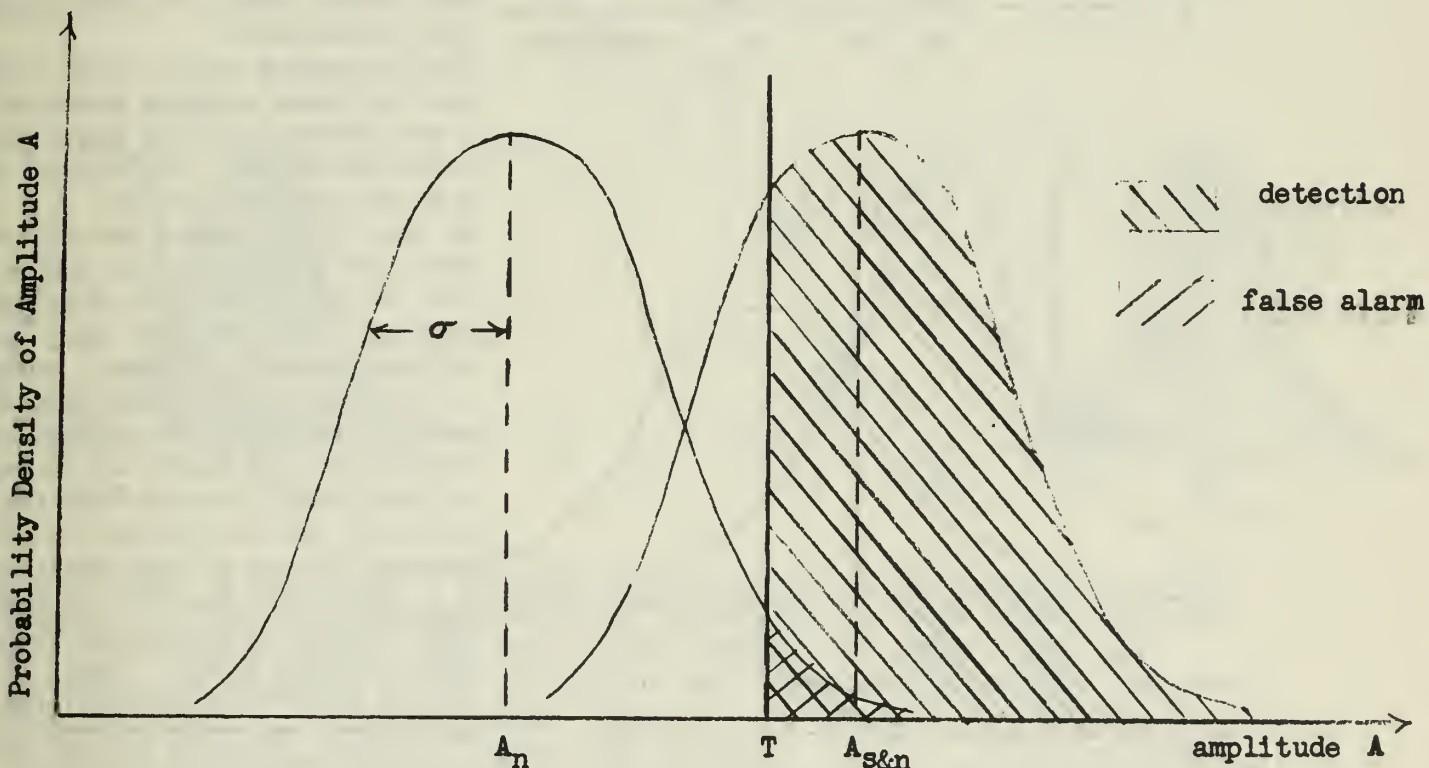
where

   A_n is the mean value of the amplitude with noise alone
   $A_{s\&n}$ is the mean value of the amplitude with signal and noise
   σ is a measure of the width of the curves (assumed equal)
   T is the threshold

The shaded area marked "detection" is the probablity of making a valid detection when the threshold is T. (The area under the curve is taken as unity.) And the shaded area marked "false alarm" is the probablity of making a "detection when the signal is absent.

Let P(D) ▪ the probability of detection

   P(FA) ▪ the probability of a false alarm

81

We need to find some simple criterion which will describe how the curves of noise alone and signal and noise combined overlap, since for a given value of threshold, it is the overlap of the curves which establishes the relative values of P(D) and P(FA).

Notice the configuration of the two curves given in the first graph. The curves are quite well-separated, in that their peaks have amplitudes whose difference is larger than the σ . These two curves could be considered well-separated.

In the second graph, each curve has the same average value of A as before, but now the σ' is twice as large. The curves now are not well-separated.

In the third graph, the curves have the same σ' as in the second, but the two have been shifted further apart so that they are well-separated. Indeed, comparison of the first and third graphs will reveal that the amount of overlap of the pairs of curves is the same. Notice that the ratio of the difference in the average values of the amplitudes $A_{s\&n}$ and A_n to the σ in the first graph is identical with that for the third. Thus, it is the separation between the two curves in units of the width of the curves which measures how well-separated the two curves are. It is thus plausible to define the detection index d by

$$d = \left(\frac{A_{s+n} - A_n}{\sigma}\right)^2$$
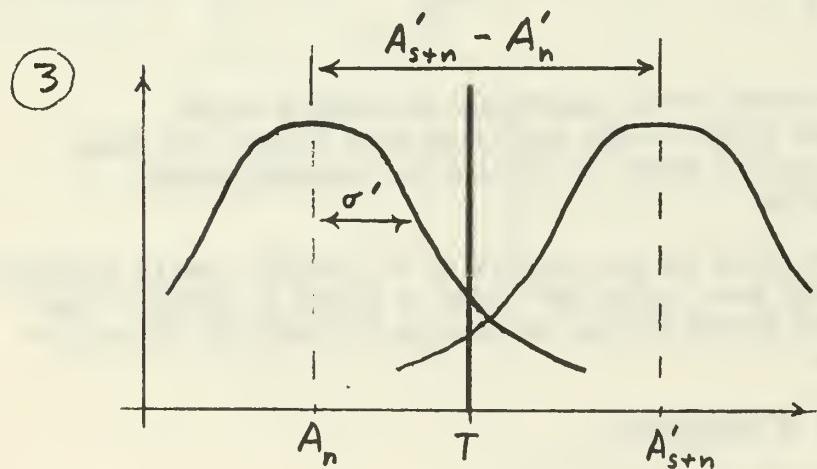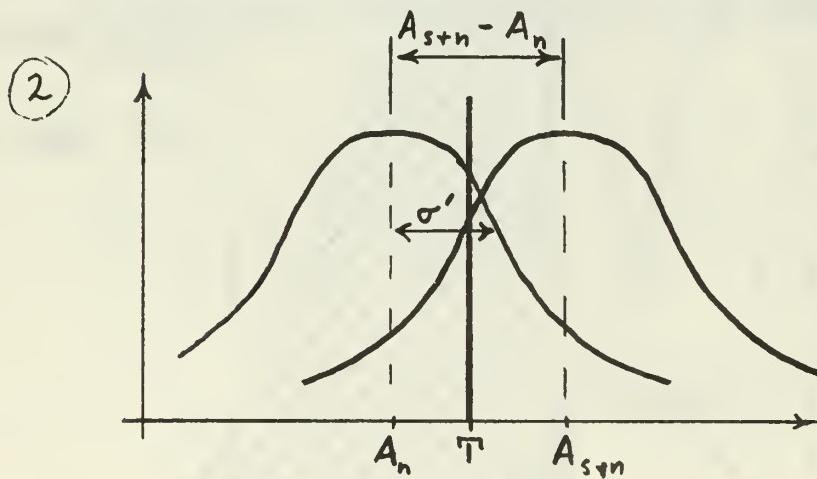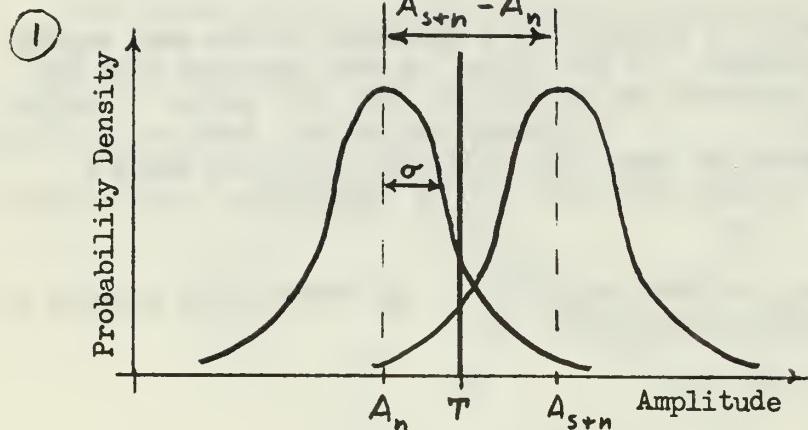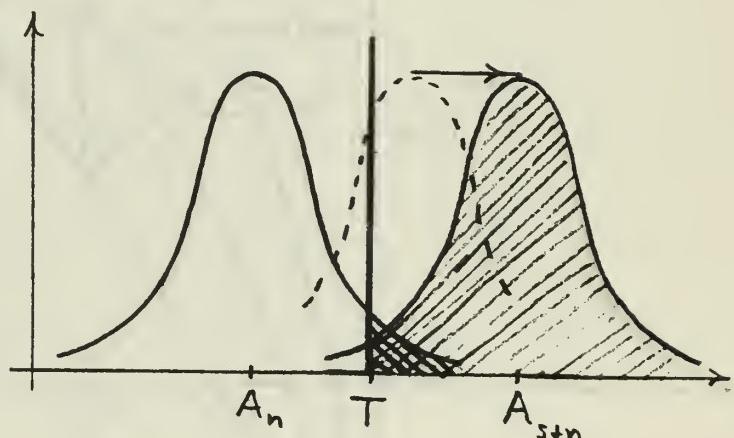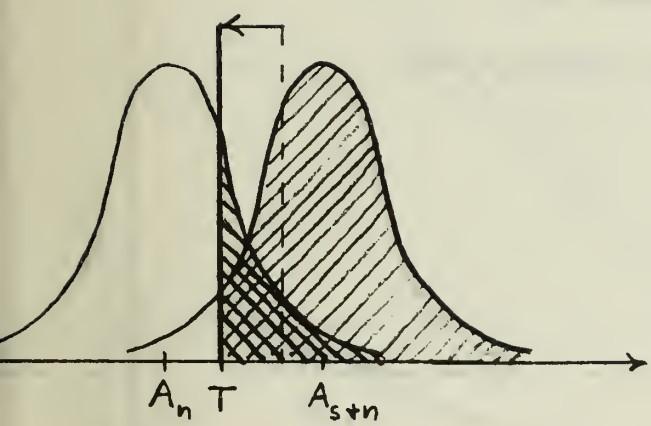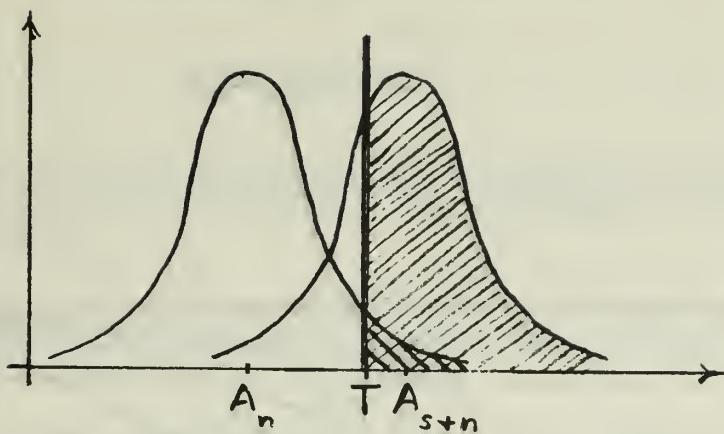
Let us study the effects of changing the positions of the threshold and the detection index on the probabilities of detection and false alarm. If the original configuration of the noise curve, signal and noise curve, and threshold are as given in the top curve, then we can establish the indicated P(D) and P(FA) by the shaded areas. Now, if the threshold is lowered, as in the second figure, then P(D) shows some increase, but the P(FA) increases rather drastically: lowering the threshold will increase the probability of a detection, but the "clutter" will also increase. On the other hand, if the noise curve and threshold are maintained constant, but the signal gets stronger, then the curve of signal and noise will move to the right. This means that the detection index increases, and the P(D) increases while P(FA) remains the same. This says no more than that a stronger signal has a greater probability of being detected.

If the statistical properties of the noise and the combined signal and noise are known, then it is possible to make curves of the P(D) and P(FA) relationships for various values of detection index. These are known as Receiver Operating Characteristics (ROC) curves.

RECEIVER OPERATING CHARACTERISITCS

ROC
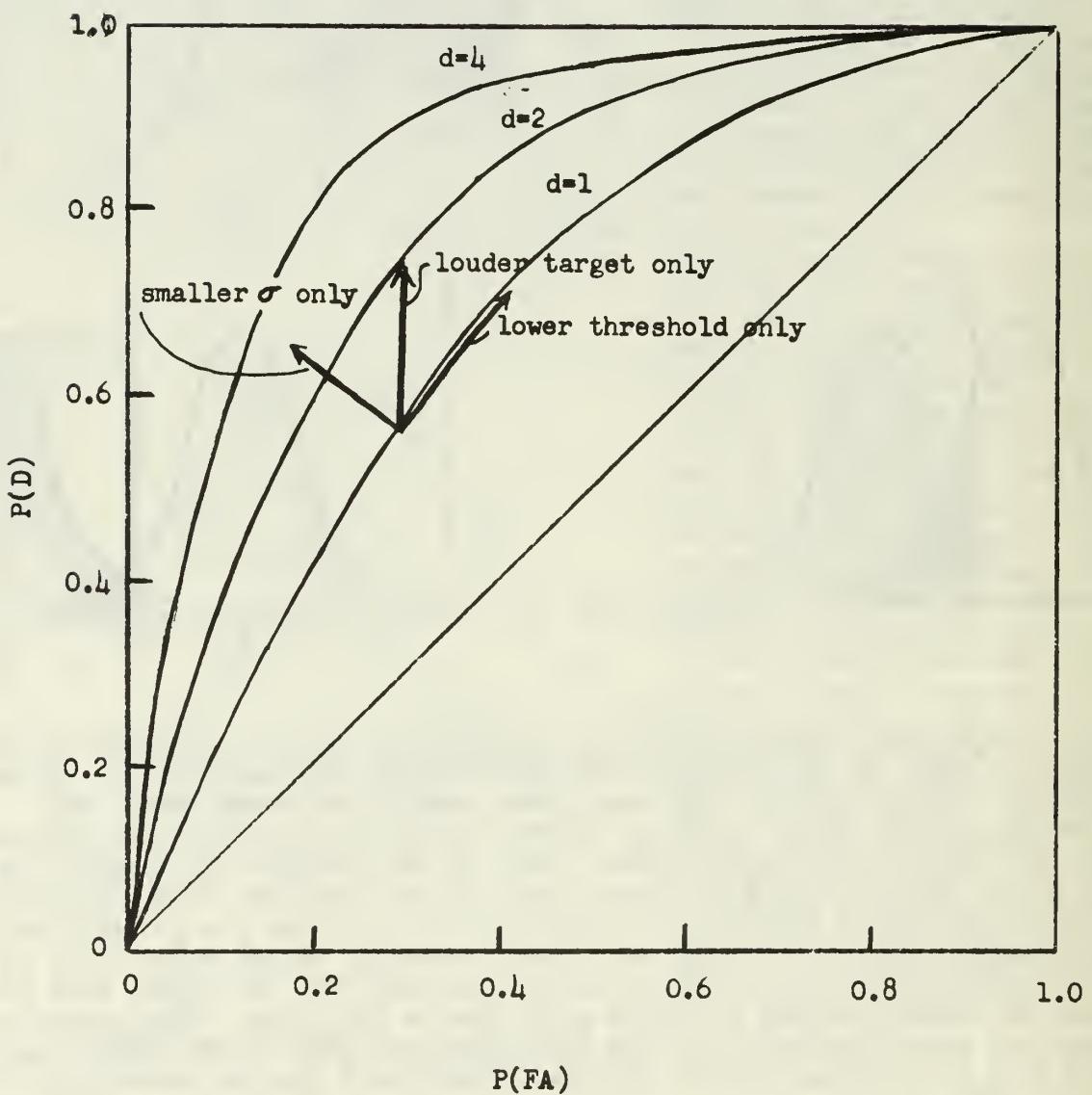
Typical ROC Curves

FALSE ALARM RATE

The rate of false alarms is the product of the sampling rate and P(FA).
Since sampling rates are generally high, the importance of a very low
P(FA) should be noted.

For example: If the output of each of 10 parallel filters is sampled
for one second each, a P(FA) of 1% will result in an average of one
false alarm every 10 seconds.

The sketch below shows a representative plot of the relationship between
d and P(FA), for a P(D) = 0.5:
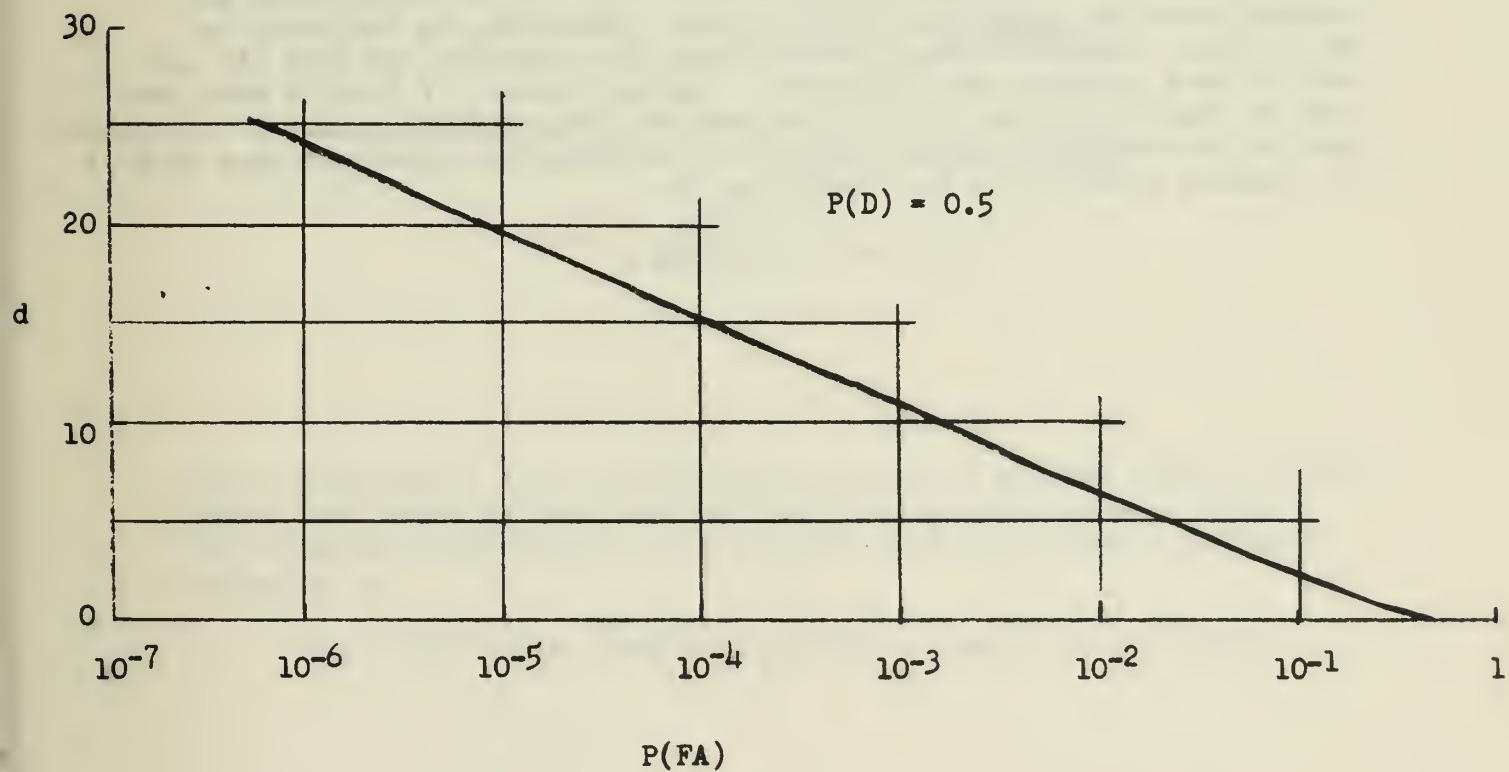
ENERGY DETECTION

For a passive detection system,
a completely unknown signal, and Gaussian noise, best performance is
obtained by filtering in a bandwidth w, taking the square of the signal,
and accumulating over a processing time t. If the signal-to-noise
ratio is small, it can be shown that the detection threshold for this
process is

$$DT_e = 5 \log (d/wt)$$

where wt is called the time-bandwidth product.

To decrease DT for a fixed d, it is necessary to increase the time-bandwidth
product.

Footnote:
     The detection threshold used in this material is that calculated over
the complete bandwidth of the processor. It often occurs that what is
specified is the detection threshold in a one Hertz bandwidth, DT(1).
This latter quantity occasionally leads to SONAR equations which have a
slightly simpler format when it is assumed that the spectrum levels are
constant over the bandwidth of the system. Otherwise, we feel that the
use of DT(1) introduces more possibilities for confusion, so that its use
creates more problems than it solves. (In particular, if Urick's excellent
book is studied, it must be fully understood that whereas he uses DT throughout
most of the text, his chapter devoted to the detection threshold uses DT(1).)
The formula relating the two quantities is

$$DT(1) = DT + 10 \log w$$

DOPPLER SHIFT

LOGS (BASE 10)

v

$\times$ DOPPLER $\times$

FREQUENCY

f

Δf

TIME
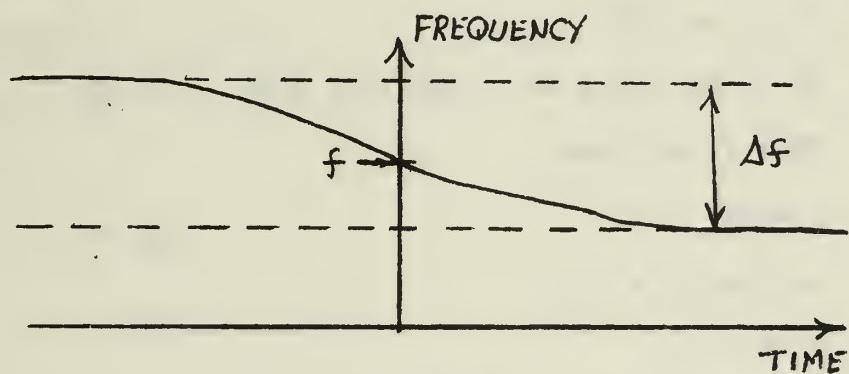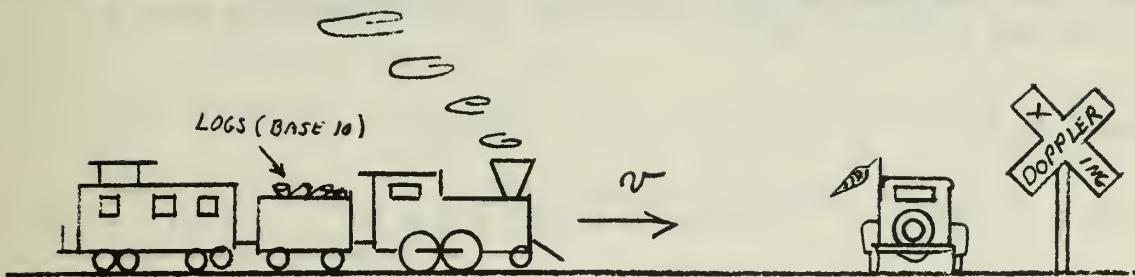
$$\Delta f = 2\frac{v}{c}f$$

While decreasing w will improve the ability of a paasive SONAR to detect
tonals, it cannot be made smaller than the expected Doppler shifts.
For a passive SONAR system, this requires that the minimum acceptable
bandwidth be

w(in Hz) = 0.7 x (frequency in kHz) x (range rate in knots)

EXAMPLE 1

A submarine with a 160 dB re 1 μ Pa line at 500 Hz crosses a convergence zone at 40 km from an omnidirectional hydrophone. The bandwidth of the receiver is 100 Hz and the sea state is 3. Assuming a convergence gain of 12 dB, find out how long the signal must be accumulated to give a P(D) = 0.50 and a P(FA) = 10^{-4}.

Solution:

$$TL = 20 \log r + ar - G = 20 \log (4 \times 10^4) + 5 \times 10^{-5} \cdot 4 \times 10^4 - 12 = 82 \text{ dB}$$

$$ML = NSL + 10 \log w - DI = 66 + 20 - 0 = 86 \text{ dB}$$

$$SL - TL \geqslant ML + DT$$

$$DT \lesssim SL - TL - ML = 160 - 82 - 86 = -8 \text{ dB}$$

$$DT = 5 \log (d/wt)$$

where $d = 15$

$\log (wt/d) = 1.6$

$wt/d = 40$

$wt = 600$

$t = 6 \text{ sec}$

EXAMPLE 2

Same as Example 1, but let w be just wide enough to encompass a 20 knt target.

Solution:

w = 0.7 x (frequency in kHz) x (range rate in knots)

= 0.7 ½ 20 = 7 Hz

ML = NSL + 10 log w - DI = 66 + 8.5 = 74.5 dB

DT < SL - TL - ML = 160 - 82 - 74.5 = 3.5 dB

DT = 5 log (d/wt)

where   d = 15

log (d/wt) = 0.7

d/wt = 5

wt = 3

t = 3/7 sec

EXAMPLE 3

A conventional submarine at periscope depth is traveling at 4 kts. The
Sea State is 3 and the layer depth is 100 m. At what range will the
submarine be detected by a 36 m deep hydrophone listening at 1000 Hz
if DI = 20 dB and DT = 0 dB?

Solution:

$$SSL \ - \ TL \ \geqslant \ NSL \ - \ DI \ + \ DT$$

$$TL \ \leqslant \ SSL \ - \ NSL \ + \ DI \ - \ DT \ = \ 120 - 62 + 20 \ = \ 78 \text{ dB}$$

For sound to be trapped in the duct,

$$f \ > \ f_{min} \ = \ 2 \cdot 10^5 / D^{3/2} \ = \ 200 \text{ Hz}$$

So we use the TL for a mixed layer

$$TL \ = \ 10 \log r_t \ + \ 10 \log r \ + \ ar \ + \ b(r/r_s)$$

$$r_t \ = \ 1.05 \cdot 10^2 \ D/(d - z_s)^{1/2} \ = \ 1.05 \cdot 10^2 (100/8) \ = \ 1310 \text{ m}$$

$$r_s \ = \ 8.4 \cdot 10^2 \ D^{1/2} \ = \ 8.4 \cdot 10^2 \cdot 10 \ = \ 8400 \text{ m}$$

From the graphs and equations

$$a \ = \ 6 \cdot 10^{-5} \ \text{dB/m}$$

$$b \ = \ 1.7 \text{ dB/bounce}$$

Notice that $\quad ar + b(r/r_s) \ + \ (a + b/r_s) \ r \ = \ 2.6 \cdot 10^{-4} \ r$

Thus $\quad 78 \ = \ 10 \log 1310 \ + \ 10 \log r \ + \ 2.6 \cdot 10^{-4} \ r$

$$10 \log r \ + \ 2.6 \cdot 10^{-4} \ r \ = \ 47$$

Now, trial and error

r	$10 \log r$	$2.6 \cdot 10^{-4} \ r$	TL
$5 \cdot 10^4$	47	13	60
$2 \cdot 10^4$	43	5.2	48
$1.5 \cdot 10^4$	41.8	3.9	46
$1.7 \cdot 10^4$	42.3	4.4	47

$$r \ = \ 1.7 \cdot 10^4 \text{ m}$$

EXAMPLE 4

Repeat Example 3 but with DI = 0 dB.

Solution:

$$TL \leqslant SSL - NSL + DI - DT = 120 - 62 = 58 \text{ dB}$$

$$TL = 10 \log r_t + 10 \log r + 2.6 \times 10^{-4} r$$

$$58 = 10 \log 1310 + 10 \log r + 2.6 \times 10^{-4} r$$

$$58 = 31 + 10 \log r + 2.6 \times 10^{-4} r$$

$$10 \log r + 2.6 \times 10^{-4} r = 27$$

r	$10 \log r$	2.6×10^{-4}	TL	
5×10^2	27	0.13	27	Bingo

But $r < r_t$

So we must use this form of the TL:

$$TL = 20 \log r + ar$$

$$58 = 20 \log r$$

$$\log r = 2.9$$

$$r = 800 \text{ m}$$

EXAMPLE 5

Repeat Example 3 with $f = 100$ Hz. Assume "heavy shipping"as would be encountered in the Mediterranean Sea.

Solution:

$$f < f_{min}$$

No trapping in the layer. Use

$$TL = 20 \log r + ar$$

$$TL \lesssim SSL - NSL + DI - DT = 133 - 76 + 20 = 77 \text{ dB}$$

$$20 \log r + 10^{-6} r = 77$$

r	$20 \log r$	$10^{-6} r$	TL	
8×10^3	78	8×10^{-3}	78	
7×10^3	77	7×10^{-3}	77	Bingo

$$r = 7 \times 10^3 \text{ m}$$

ACTIVE SONAR
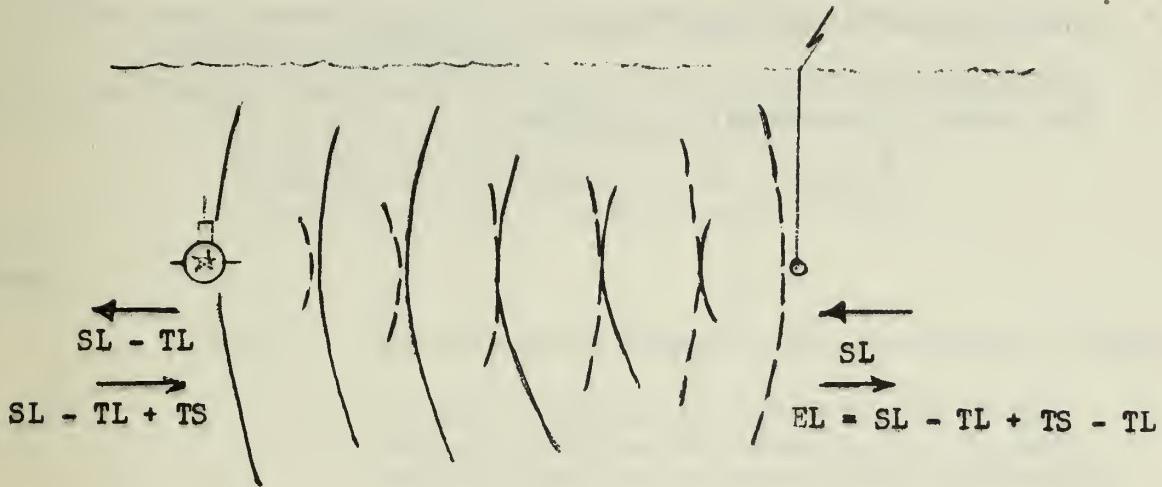
$$SL - 2TL + TS \geqslant ML + DT$$

where

      SL = source level of transmitter

     2TL = two-way transmission loss

     TS = target strength

Each term in the above equation will now be discussed separately.

Note: the above active SONAR equation is for a <u>monostatic</u> case, wherein the source and receiver are coincident so that the TL from source to target is the same as the TL from target to receiver. Our discussion will usually be restricted to this case. Extension to the <u>bistatic</u> situation in which source and receiver are at different locations is trivial: If the transmission loss from source to target is TL, and from target to receiver TL', then all that is required is to replace 2TL with TL + TL', so that we have

$$SL - TL - TL' + TS \geqslant ML + DT$$

LIST OF ADDITIONAL SYMBOLS FOR ACTIVE SONAR

SL = source level of the active sonar system

TS = target strength of the desired target

RL = reverberation level

TS' = target strength of the scatterers causing reverberation

TL = transmission loss from source to target

TL' = transmission loss from target to receiver, equal to TL in the case of monostatic systems

L_e = "effective length" of a target

t = pulse duration of the acoustical signal from the source

s_v = a measure of the scattering cross-section per unit volume

s_s = a measure of the scattering cross-section per unit area

S_v = scattering strength for volume scatterers

S_s = scattering strength for surface scatterers.

$$\boxed{SL} \ - \ 2TL \ + \ TS \ \rangle \ ML \ + \ DT$$

SOURCE LEVEL - ACTIVE SONAR

SL = sound pressure level 1 m from the acoustic center of the transmitting transducer.

$$SL \ = \ 10 \ \log \ (\text{Power}) \ + \ DI \ + \ 171$$

where

Power = Acoustic power output of the transducer in Watts

DI = 10 log (the ratio of the intensity on the acoustic axis to the intensity at the same distance from an omnidirectional transmitter with the same acoustic power output).

or

$$DI \ = \ 20 \ \log \ P(r)/P_0(r)$$

where $P_0(r)$ = pressure produced by an omnidirectional transducer with the same power output

and

171 = factor to take care of the units

Acoustic power output (Watts)
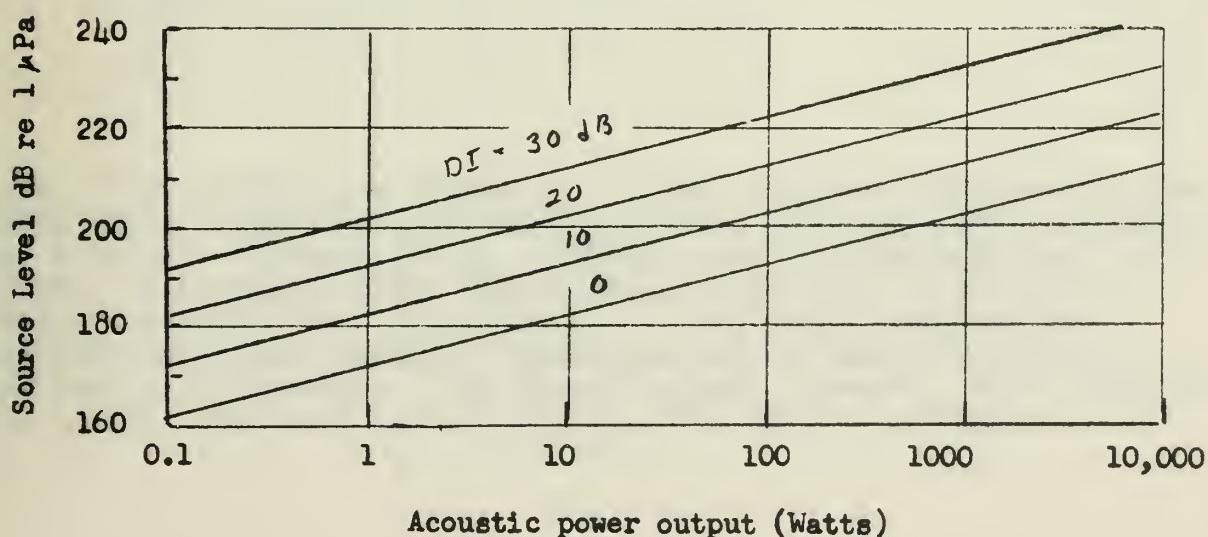
$$SL - 2TL + \boxed{TS} \geqslant ML + DT$$

TARGET STRENGTH

TS = (sound pressure level of reflected wave 1 m from the acoustic center of the target in a direction back towards the receiver) - (sound pressure level of the incident wave at the position of the target if the target were absent).

or

TS = 20 log P'(r'=1)/P(r)

where

P'(r'=1) is the pressure of the reflected wave 1 m from the acoustic center of the target in a direction back towards the receiver.

Target Strength depends on shape, construction, and orientation of the target, and on the frequency and pulse length of the sound wave.

effective length L_e

Short Pulse

$|\!\leftarrow ct \rightarrow\!|$

Tone Burst

$|\!\leftarrow\!\!\!-\;\; ct + 2L_e \;\;-\!\!\!\rightarrow\!|$
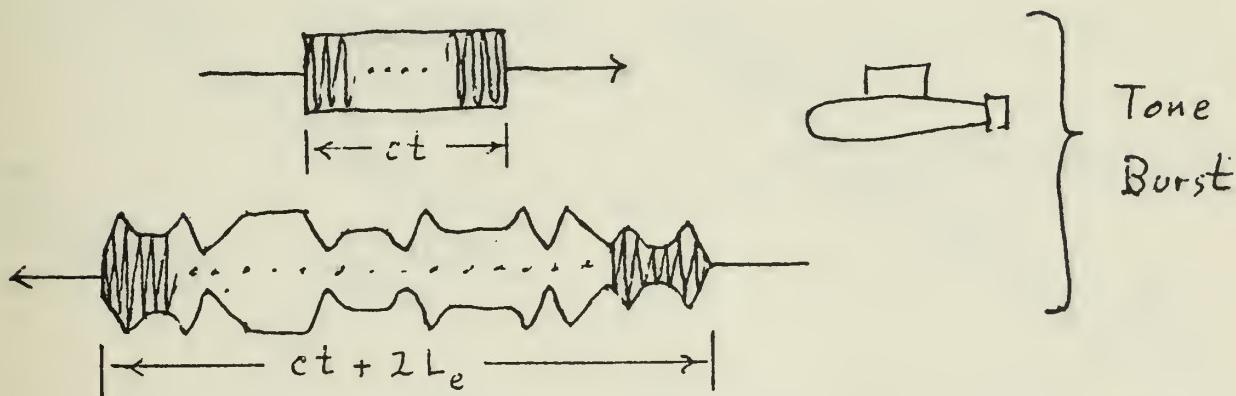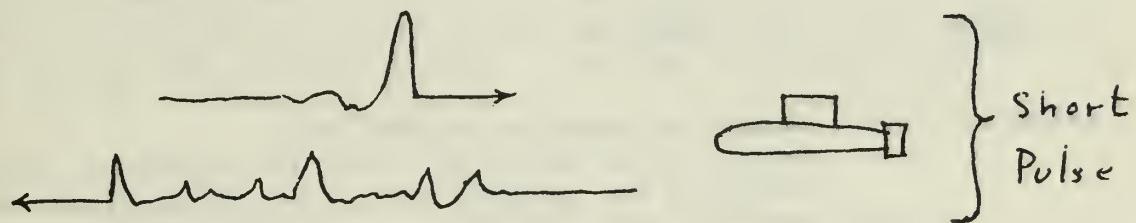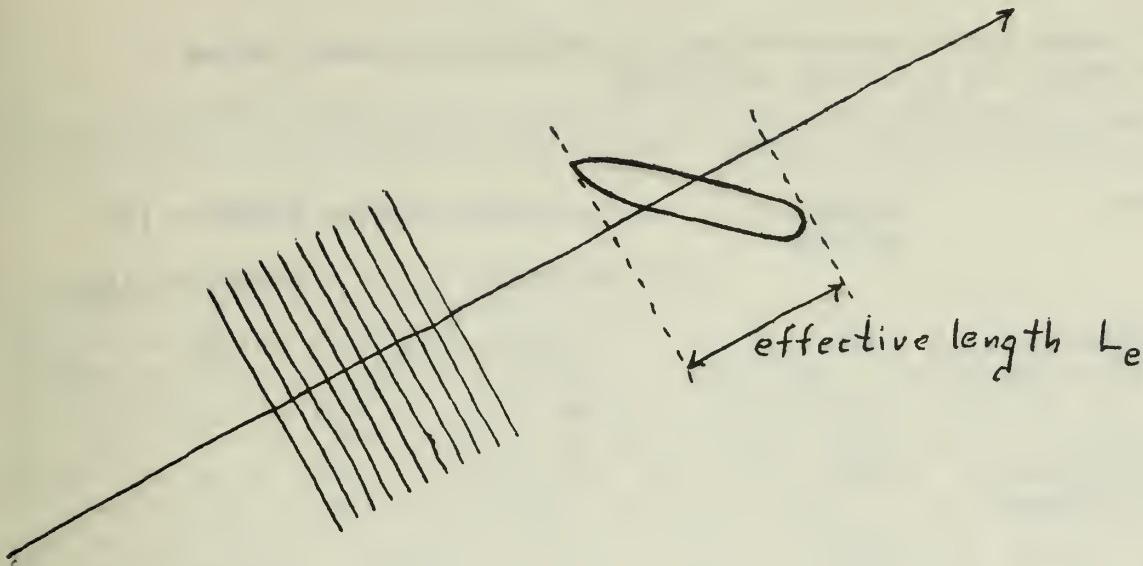
Because of the effective length L_e of the target, portions of the incident pulse which reflect from the tail (in the above sketches) travel a total distance of $2L_e$ further than the reflections from the nose. This means that the apparent duration of the pulse is stretched from t to $(t + 2L_e/c)$. For very long pulses, the additional time interval corresponding to the round trip, $2L_e/c$, can be neglected. For short pulses, however, this stretching can be quite important in determining the target strength.

The Table shows a few representative, but very approximate, values of target strength for a variety of objects,

Target	Frequency in kHz	Approximate Target Strength (dB)	
		Beam Aspect	Bow-Stern Aspect
Fleet Submarine (approx 100 m long)	10	35	30
	24	25	16
S Class Submarine (approx 70 m long)	24	20	13
Torpedo	24		-20
Surface Vessel		10 to 30	
3 ft mine	30-90	-8	

Extracted from: Physics of Sound in the Sea, NDRC
Principles of Underwater Sound for Engineers, Urick
Principle and Applications of Underwater Sound, NDRC
NAVSHIPS 0967-129-3010

$$SL - 2TL + TS \geqslant \boxed{ML} + DT$$

MASKING LEVEL

The undesired noise competing with the desired signal can arise from two possible mechanisms in the case of active SONAR:

Masking from ambient or self noise

Masking from reverberation

Which of these two mechanisms is the dominant depends on the range between source and target. Generally speaking, for low-powered active systems the performance will be noise-limited, in that the maximum effective detection range will be determined when the echo level diminishes below that level for which it can be extracted from the ambient or self noise. For higher-powered active systems, however, reverberation becomes an important source of interference. Reverberation tends to decrease with increasing range, but less rapidly than the echo level. Thus, if the echo level has diminished until it is buried in the reverberation, but the reverberation has not yet fallen sufficiently below the ambient (or self) noise, then the maximum detection range is determined by the reverberation. The two situations are suggested in the figures:
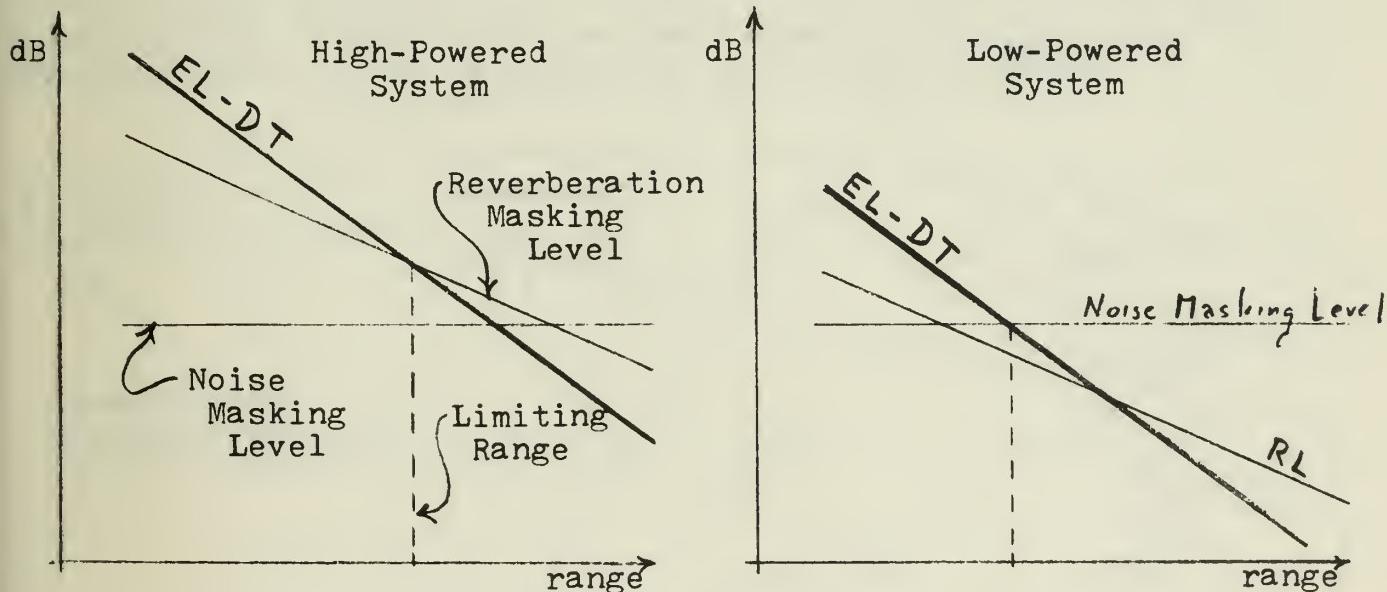
The choice is a little more subtle than suggested by the simple curves above. The detection threshold for noise masking may be different than that for reverberation masking, and the detection threshold for reverberation-limited performance may depend strongly on the doppler-induced difference between the frequency of the reverberant sound and that reflected from the desired target.

AMBIENT AND SELF NOISE

The masking level is given by

$$ML = NL - AG$$

where the noise level NL is that from the ambient noise of the ocean, noise generated by the receiving platform, or the combination of the two if they are of nearly equal strengths.

Recall that if the noise spectrum level is sufficiently smooth over the receiver bandwidth, then we can write

$$NL = NSL + 10 \log w \quad .$$

Otherwise, the total bandwidth must be broken down into smaller bands, the band level found for each, and then these band levels combined according to the nomogram for combining independent levels.

REVERBERATION

If the sound field generated by the source is strong enough, and
there are enough scatterers present in the vicinity of the desired
target, then unwanted reflections and scattering of sound from
the undesired scatterers are received along with the echo from
the desired target. The echo level of the signal received from
the desired target is

$$EL = SL - 2TL + TS \qquad \text{(monostatic)}$$

where TS is the target strength of the desired target. The
<u>reverberation level</u> RL can be written in the form

$$RL = SL - 2TL + TS' \qquad \text{(monostatic)}$$

where TS' is the combined target strength of all the undesired
targets which are generating the masking level. Since the RL
is the masking level when reverberation is the dominant source
of unwanted sound, the SONAR equation becomes

or
$$\boxed{\begin{aligned} EL - RL &\geqslant DT \\ TS - TS' &\geqslant DT \end{aligned}} \qquad \text{(monostatic)}$$

The central problem in obtaining TS' is determining the volume
or surface area containing the scatterers which is illuminated
by the source, sending sound to the receiver, and sending the
sound so that it arrives at the same time as the signal from the
desired target. This is suggested in the sketch below.

Source
SL

TL

Target TS

Reverberation-
Limited Performance
(bistatic)

TL'

Reverberating
Region
TS'

Receiver

$EL = SL - TL - TL' + TS$

$RL = SL - TL - TL' + TS'$
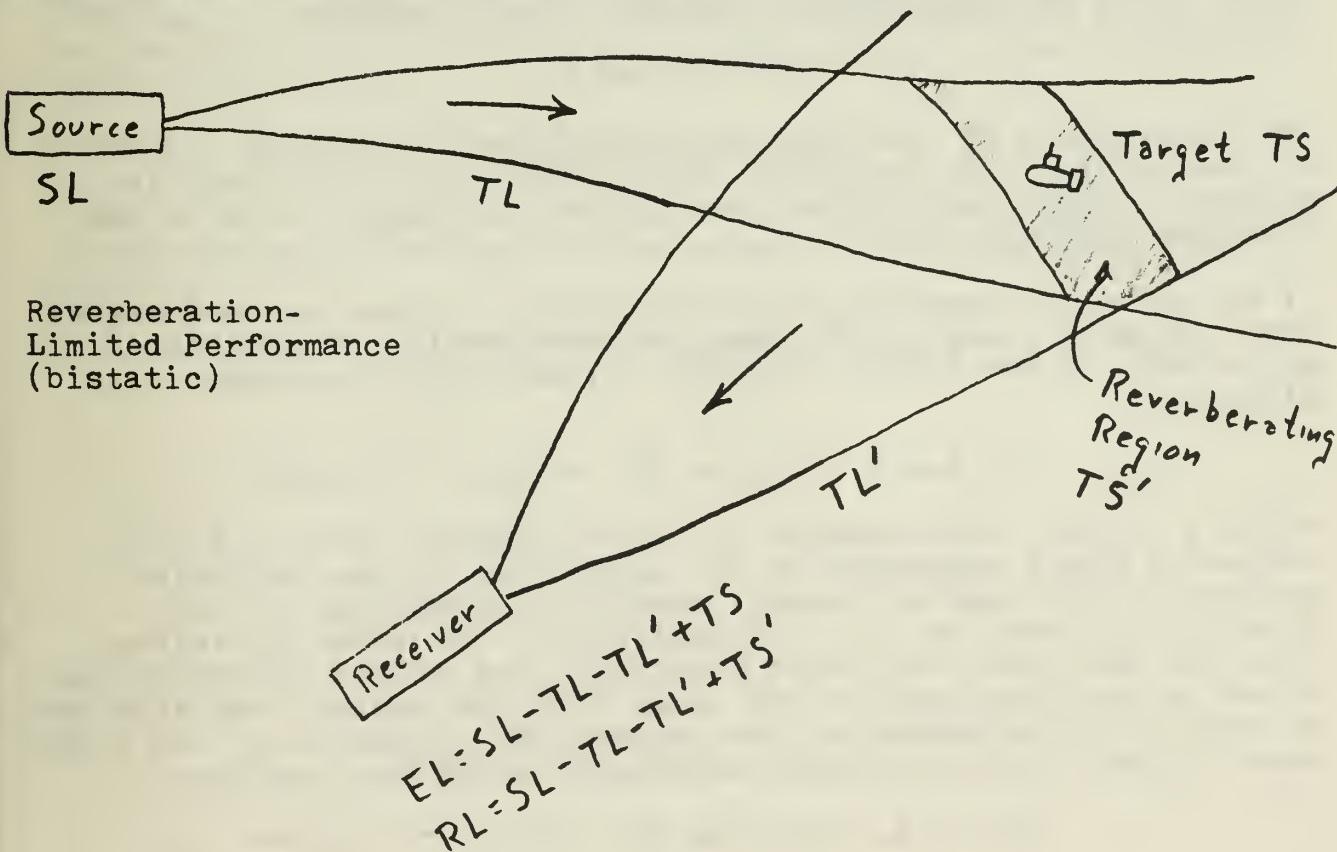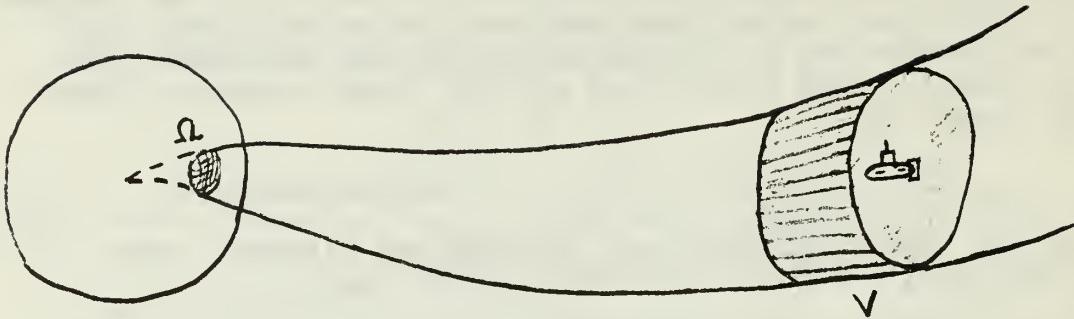
The target strength TS' of the undesired scatterers can be
written as

$$TS' = 10 \log s_v V$$

where V is the volume within which the reverberation sent to
the receiver is in competition with the reflected signal from
the desired target and s_v is a measure of the scattering cross
section per unit volume. It has become conventional to express
s_v in dB terms through the definition

$$S_v = 10 \log s_v$$

where S_v is the <u>scattering strength</u>. Thus, we have

$$TS' = S_v + 10 \log V.$$

The calculation of the <u>reverberating volume</u> V contains the heart
of the problem. As suggested previously, this is a study in
geometry, involving the way in which major lobes of source and
receiver overlap.

If we restrict ourselves to the monostatic situation, and assume
that the major lobe of the receiver completely overlaps the
major lobe of the source, it then follows that the volume V is
given by

$$10 \log V = 10 \log \frac{ct}{2} + TL_g + 10 \log \Omega$$

where t is the time duration of the acoustical pulse and Ω is
the solid angle subtended by the major lobe of the radiation
pattern of the source. Recall that this is related to the
directivity index by $DI \sim 10 \log(4\pi/\Omega)$. The term TL_g arises
from the fact that the cross sectional area of the reverberating
volume clearly depends on the range from the source, and this can
be seen to be expressed by the geometrical spreading of the sound
beam. Thus, for this simple monostatic situation, we have

$$TS' = S_v + 10 \log \frac{ct}{2} + TL_g + 10 \log 4\pi - DI$$

In the case of scattering which arises from a surface (such as the sea surface or the bottom) or from some well-defined layer such as the bubble layer underlying the water-air interface, the reverberation target strength is expressed as

$$TS' = S_S + 10 \log A$$

where A is the surface area generating reverberation in competition with the desired target and S_S is the <u>scattering strength</u> (per unit area).

As before, the problem is one primarily of geometry, involving the way in which source and receiver major lobes overlap. If we again restrict ourselves to the simple monostatic geometry used previously (for which the major lobe of the receiver completely overlaps that of the source), then the surface area of interest can be seen to be

$$A = \tfrac{1}{2} r \, \theta \, c \, t$$

where θ is the angle in radians of the horizontal angular width of the radiation pattern of the source. This then gives the equation

$$TS' = S_s + 10 \log \frac{ct}{2} + 10 \log r + 10 \log \theta$$

REPRESENTATIVE SCATTERING STRENGTHS

The values quoted here are representative only. Large deviations can occur depending on location, season, biological activity, and so forth.

> Deep Scattering Layers: The scattering strength S_v can vary between about -90 to -60 dB within the various sublayers.

> Water Volume: In the clear open ocean waters, S_v can be found to range between -100 to -70 dB.

> Surface Scattering: For reasonably low grazing angles, the combined scattering from the ocean surface and the population of bubbles found at shallow depths below the surface gives scattering strengths S_s ranging around -50 to -30 dB for sea states between 1 and 4.

> Bottom Scattering: For grazing angles between 20° and 60°, representative values of the scattering strength S_s range between -40 and -10 dB. The exact values depend very strongly on the particular bottom and its composition. In general, however, as the grazing angle goes to zero, the scattering becomes negligible and S_s goes to $-\infty$ dB.

REVERBERATION LEVEL DEPENDENCIES

Notice that for either volume or surface reverberation the reverberation target strength TS' increases with increasing range, either as TL_g (volume) or 10 log r (surface). This is to be compared with ambient noise masking, for which there is no range dependence. This means that, given sufficient source level, as SL increases SONAR performance will change from <u>ambient-noise limited performance</u> to <u>reverberation limited performance</u>. While detection for ambient noise limited performance depends on the source level and directivity index, notice that when the performance becomes reverberation limited there is no dependence on the source level: additional power will <u>not</u> improve reverberation limited performance. Reverberation can be reduced by increasing the directivity of the source and/or receiver, or by decreasing the pulse length, given by ct. Increased directivity has the disadvantage of reducing scanning rates, and decreased pulse length may result in reduction of the target strength TS of the desired target.

DOPPLER SHIFT FOR ACTIVE SONAR

In the case of active SONAR, any motion of the platform containing the source will cause the signal propagated into the water to have a frequency shift which depends on the bearing into which the signal is sent. This doppler-shifted signal is subjected to a further doppler-shift when it is reflected from the moving target. Without going into the specifics of the various frequency shifts, we shall simply quote some results of interest:

Because of the two-way propagation of acoustical energy from the moving source to the moving target and then reflection back to the moving receiver, the required bandwidth for the receiver of an active sonar operated under monostatic conditions is

$$w \text{ in Hz} = 1.4 \cdot (\text{Range rate in knots}) \cdot (\text{Frequency in kHz})$$

Notice that this is twice the bandwidth required in the passive SONAR case.

If the target is stationary with respect to the surrounding water, then the desired echo will have the same frequency as the reverberation and the ability of the receiving system to recover signal from noise will be minimized. On the other hand, when the target is moving through the water, then the echo and reverberation will have different frequencies and detection will be much easier. This effect is quite easy to observe by ear: The detectibility of an "up-doppler" or "down-doppler" echo in reverberation is very easy even when the relative loudness of the desired echo is rather weak. When there is no frequency change between echo and reverberation, however, the ability of the ear to detect the echo is drastically reduced. The strong increase of the detection threshold for slow targets points to the use of parallel filters of relatively narrow bandwidths to facilitate the separation of signal from reverberation when the difference in frequency is small.

$$SL - 2TL + TS \geqslant ML + \boxed{DT}$$

DETECTION THRESHOLD

Energy Detection

If, as was the case for the passive SONAR equations, the mechanism for the detection of the desired signal is based on detecting an excess of acoustical energy in the frequency band of interest, then we have the same expression for the detection threshold,

$$DT = 5 \log \frac{d}{wt}$$

where t is the integration time. In the case of active sonar, t should be the length of the received echo. If the integration time is less than or greater than this pulse duration, it can be shown that the DT is increased above this expression, and performance degraded.

Correlation Detection

An alternative mode of signal processing is possible in the case of active SONAR. Since the amplitude and frequency properties of the tone burst generated by the source are known, it is possible to search for a signal of these same properties in the received echo. If the detailed shape of the received echo matches that of the sent pulse, then it can be shown that the detection threshold is given by

$$DT = 10 \log \frac{d}{2wt}$$

when the processing is that of correlation of the received signal against a replica of the generated pulse. There are many modifications of this basic idea, but all rely on the technique of multiplying the received signal by a time-delayed model of the sent pulse and integrating the resultant product over the pulse duration. This is done for monotonically increasing delay times, and the resulting function of delay time studied for the presence of a sharp peak representing the matching of the delayed replica with the same signal appearing in the received signal and noise. In practice, the finite size of the target, multipath interference, and the fluctuations introduced by the inhomogeneities of the transmission of the signal through the water alter the received echo so that the correlation process is not as good as would be desired. As a result, the DT for a real system operating in the real world is more than the theoretical expression would predict.

SUMMARY OF THE ACTIVE SONAR EQUATIONS

Ambient noise limited conditions

$$SL - 2TL + TS \geqslant NSL - AG + 10 \log w + DT$$

Reverberation limited conditions

$$TS - TS' \geqslant DT$$

An active SONAR operates at 1 kHz with a source level of 220 dB re 1 μPa, a directivity index of 20 dB, a horizontal beam width of 10°, and a pulse length of 0.1 sec. Correlation detection is used and it is desired to have P(D) = 0.50 with P(FA) = 10^{-4}. The target strength of the submarine is expected to be 30 dB and its speed may be up to 20 kts. Both the SONAR and the submarine are in a mixed layer of depth 100 m with the source at 36 m. The sea state is 3, the scattering strength for surface scatterers is -30 dB, and volume scattering is negligible. For simplicity, assume that conditions are such that the detection threshold is the same for both noise-limited and reverberation-limited performance.

Solution:

w = 1.4 (frequency in kHz) (range rate in kts) = 1.4 x 1 x 20 = 28 Hz

d = 15   same calculation as in example 1 (passive)

DT = 10 log (d/2wt) = 10 log (15/2x28x0.1) = 10 log 2.68 = 5 dB

NL = NSL - DI + 10 log w = 62 -20 + 14 = 56 dB

r_t = 1310 m   and r_s = 8400 m   same calculation as Example 3 (passive

Assume that the detection is noise limited

2TL $\leqslant$ SL + TS - NL - DT = 220 + 30 -56 - 5 = 189

TL $\leqslant$ 95

TL = 31 + 10 log r +2.6 x 10^{-4} r   same calculation as Example 3

10 log r + 2.6 x 10^{-4} r = 95 - 31 = 64

r	10 log r	2.6 x 10^{-4} r	TL
10^4	40	26	43
10^5	50	26	76
5 x 10^4	47	13	60
6 x 10^4	48	16	64

r = 6 x 10^4 m   if noise limited

EXAMPLE (CONTINUED)

But, what is the RL at this range?

$$TS' = S_s + 10 \log (ct/2) + 10 \log r + 10 \log \theta$$

$$= -30 + 10 \log (1500 \times 0.1/2) + 48 + 10 \log 0.174$$

$$= -30 + 19 + 48 - 8 = 29 \text{ dB}$$

$$RL = SL - 2TL + TS'$$

$$= 220 - 189 + 29 = 60 \text{ dB}$$

So

$$RL > NL$$

Therefore the detection will be <u>reverberation</u> limited

$$TS - TS' \geqslant DT$$

$$TS' = -30 + 19 + 10 \log r - 8 = -19 + 10 \log r$$

$$TS - TS' \geqslant DT$$

$$30 + 19 - 10 \log r \geqslant 5$$

$$10 \log r \leqslant 44$$

$$r = 2.5 \times 10^4 \text{ m}$$

APPENDIX A: LOGS

All you have ever wanted to know about the decibel but were afraid to ask.
The language in which the SONAR equation is written is the language of the decibel. This self-paced exercise is intended to make you familiar with the grammar of this language.

Instructions. For each numbered section, first attempt to do the problems. (The answers are on the last page.) If you have no troubles, skip over to the next section. On the other hand, if you have difficulties, read through the discussion and then try the problems again. Go to the next section only when you can do the problems without difficulty. By the time you finish all the sections, you should be speaking "decibel" like a native.

1. EXPONENTIAL NOTATION

Problems A

1. Write each of the following numbers in exponential forms

    a)   2,365,000

    b)   872

    c)   3340

    d)   42.1

    e)   600

    f)   1.0

2. Write each of the following numbers in conventional form

    a) 2.689×10^2

    b) 7.30×10^2

    c) 6.29×10^4

    d) 8.9×10^6

    e) 3.261×10^2

    f) 1.00×10^0

3. Write each of the following numbers in exponential form

   a) 0.00252

   b) 0.012

   c) 0.00002

   d) 0.01002

   e) 0.00328

4. Write each of the following numbers in conventional form

   a) 1.03×10^{-6}

   b) 3.2×10^{-2}

   c) 6.89×10^{-4}

   d) 1.003×10^{-1}

   e) 1.00×10^{-1}

A succinct way of expressing very large and very small numbers is by exponential (or scientific) notation. For example, one-hundred and twenty-six million can be written either as

$$126,000,000$$

or as

$$1.26 \times 10^{8} ,$$

where the exponent 8 denotes the number of places that the decimal must be moved to the right to express the number in its conventional form.

There is no unique way of expressing a number in exponential form, since

$$1.26 \times 10^{8} = 12.6 \times 10^{7} = 0.126 \times 10^{9} = \text{etc.}$$

Which form you use depends on convenience.

Numbers smaller than unity can be handled in a similar manner. For example, one-hundred and twenty millionths can be written either as

$$0.000120$$

or as

$$1.2 \times 10^{-4} ,$$

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where the negative exponent denotes that the decimal place is to be moved to the <u>left</u>.

A special case that must be watched is

$$1 = 1 \times 10^0 = 10^0$$

which is consistent with the above conventions.

2. <u>ALGEBRA OF EXPONENTS</u>

Problems B

1. Calculate the following and express the answer in exponential form

   a) $6.15 \times 10^3 + 2.34 \times 10^3$

   b) $3.2 \times 10^2 - 1.46 \times 10^3$

   c) $9.1 \times 10^4 + 2.4 \times 10^5$

   d) $3.58 \times 10^{-2} + 1.26 \times 10^{-2}$

   e) $6 \times 10^{-6} + 3.281 \times 10^{-4}$

   f) $3.5 \times 10^4 - 2 \times 10^1$

2. Calculate the following and express the answers in exponential form

   a) $2.50 \times 10^6 \times 2.00 \times 10^4$

   b) $4.40 \times 10^4 / 2.20 \times 10^5$

   c) $3.26 \times 10^3 \times 2.00 \times 10^{-2}$

   d) $5.34 \times 10^{-2} / 2.67 \times 10^5$

   e) $8.68 \times 10^{-1} / 4.34 \times 10^{-2}$

3. Calculate the following and express the answer in exponential form

   a) $(1.1 \times 10^4)^2$

   b) $(2.0 \times 10^{-3})^4$

   c) $(1.44 \times 10^2)^{1/2}$

   d) $(1.44 \times 10^{-2})^{1/2}$

   e) $(8.0 \times 10^6)^{1/3}$

   f) $(2.56 \times 10^4)^{0.5}$

   g) $(8 \times 10^{-6})^{0.3333....}$

   To add and subtract numbers expressed in exponential form it is necessary
to express both numbers in the same exponent. For example, adding 3.56×10^4
to 2.1×10^3 is accomplished by

$$
\begin{array}{r}
3.56 \times 10^4 \\
\underline{0.21 \times 10^4} \\
3.77 \times 10^4
\end{array}
$$

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When asked to subtract 8.67×10^{-1} from 2.23×10^3, note that the answer is 2.23×10^3, for the term to be subtracted affects the larger term beyond the figures retained.

Exponential notation really shows its worth in calculations involving multiplication and division. We see that

$$120 \times 2000 = 240,000$$

or

$$1.2 \times 10^2 \times 2 \times 10^3 = 2.4 \times 10^5$$

and

$$36000 / 200 = 180$$

or

$$3.64 \times 10^4 / 2 \times 10^2 = 1.8 \times 10^2.$$

The rules are simple:

In multiplication, exponents add.

In division, exponents subtract.

A special case that is often useful is

$$1 / 10^2 = 10^0 / 10^2 = 10^{-2},$$

which allows exponents to be moved between numerator and denominator.

Power and roots are trivial in exponential notation:

$$(1.2 \times 10^3)^2 = 1.2 \times 10^3 \times 1.2 \times 10^3 = 1.44 \times 10^6,$$

or for any power n

$$(10^m)^n = 10^{mn}$$

and,

$$(1.44 \times 10^6)^{1/2} = (1.44)^{1/2} \times (10^6)^{1/2} = 1.2 \times 10^3,$$

or for any power of n

$$(10^m)^{1/n} = 10^{m/n}.$$

This latter form introduces the possibility of fractional exponents; these will be discussed in Section 4.

Problems C

1. Find x

    a) $\log 10^3 = x$

    b) $\log 10^{-6} = x$

    c) $\log 10^x = 4$

    d) $\log 10^x = -3$

    e) $\log 10^x = 0$

    Since we say that "ten to the fourth power equals ten thousand" we can ask "to what power must ten be raised to get ten thousand?" The answer is obviously "four." You are now an expert on logarithms, since the "log" of 10,000 is 4. This is written

$$\log 10{,}000 = 4 \text{ or } \log 10^4 = 4$$

and this is all there is to logs.

    Find. the log of x by asking yourself to what power must I raise 10 to get x.

    It is equally appropriate to ask "the log of what is 4," that is what is x so that $\log 10^x = 4$.

4. DECIMAL EXPONENTS

PROBLEMS D

1. Find

   a) log 4

   b) log 40

   c) log 0.4

   d) log 4^2

   e) log 1/4

2. Find the number whose log is

   a) 0.70

   b) 1.70

   c) -1.30

   d) -1.70

   e) -2.70

So far we have assumed integer exponents, but everything we have
said also is true for decimal exponents. For example,

$$10^{0.30} = 2.0$$

and

$$\log 2 = 0.30$$

The table below will allow you to calculate, with a fair degree of accuracy,
all logs.

X	1.0	1.2	1.5	1.7	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
log x	0.00	0.08	0.18	0.23	0.30	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1.0

If you ever need more accuracy you will have to use slide rule, calculator, or tables.

To find the log of a number greater than 10 proceed as follows:

$$\log 7000 = \log 7 \times 10^3 = \log 7 + \log 10^3$$

$$= 0.85 + 3 \quad = 3.85$$

or

$$\log 0.003 = \log 3 \times 10^{-3} = \log 3 + \log 10^{-3}$$

$$= 0.48 - 3 \quad = -2.52$$

There is one property of logs that, if used correctly, will greatly simplify many calculations. This will be illustrated by an example:

$$\log 36 = \log 6^2 = \log (6 \times 6) = \log 6 + \log 6 = 2 \log 6 = 1.56$$

(Calculate log 36 and compare to 2 log 6.)

In general

$$\log x^n = n \log x.$$

It is also true that

$$\log xy = \log x + \log y.$$

$$\log 36 = \log (4 \times 9) = \log 4 + 2 \log 3 = 0.60 + 0.96 = 1.56$$

But, remember that

$$\log (x + y) \text{ \underline{is not equal} to } \log x + \log y \text{ !!!}$$

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5. THE DECIBEL

PROBLEMS E

1. Express these signal-to-noise ratios in dB

    a) 2

    b) 1/2

    c) 10

    d) 1/10

    e) 100

2. The voltage of a signal is 1 volt; express the signal level with reference to the following voltages

    a) 10^{-12}

    b) 10^{-6}

    c) 1

    d) 10^{6}

    e) 10^{12}

By convention, when the log of a pressure or voltage is multiplied by 20, the resulting number is said to be measured in "decibels", donated by dB.

$$20 \log 2 = 6dB$$

$$20 \log 10 = 20dB$$

Decibels are used in two different ways:

a) to express a <u>ratio</u> such as the signal-to-noise ratio

signal-to-noise ratio = 20 log (signal voltage/noise voltage)

b) to express a <u>level</u> such as signal level

signal level = 20 log (signal voltage/reference voltage)

In the latter case it is absolutely essential to specify the reference. e.g., "the signal level is 120dB re 10^{-6} volts".

ANSWERS

Problems A: 1.a) 2.365×10^6, b) 8.72×10^2, c) 3.34×10^3, d) 4.21×10^1,

e) 6×10^2, f) 1.0×10^0

2.a) 268.9, b) 730, c) 62,900, d) 8,900,000, e) 326.1

f) 1.00

3.a) 2.52×10^{-3}, b) 1.2×10^{-2}, c) 2×10^{-5}, d) 1.002×10^{-2}

e) 3.28×10^{-3}.

4.a) 0.00000103, b) 0.032, c) 0.000689, d) 0.1003, e) 0.100

Problems B: 1.a) 8.49×10^3, b) -1.14×10^3, c) 3.3×10^5, d) 4.84×10^{-2}

e) 3.34×10^{-4}, f) 3.5×10^4.

2.a) 5.00×10^{10}, b) $2.00 \ 10^{-1}$, c) 6.52×10^1, d) 2.00×10^{-7},

e) 2.00×10.

3.a) 1.2×10^8, b) 1.6×10^{-11}, c) 1.2×10^1, d) 1.2×10^{-1},

e) 2.0×10^2, f) 1.6×10^2, g) $2. \times 10^{-2}$.

Problems C: 1.a) 3, b) -6 ,c) 4, d) -3, e) 0

Problems D: 1.a) 0.60, b) 1.60, c) -0.40, d) 1.20, e) -0.60.

2.a) 5.0, b) 50, c) 0.050, d) 0.020, e) 0.0020

Problems E: 1.a) 6dB, b) -6dB, c) 20dB, d) -20dB, e) 40dB.

2.a) 240dB, b) 120dB, c) 0dB, d) -120dB, e) -240dB.

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